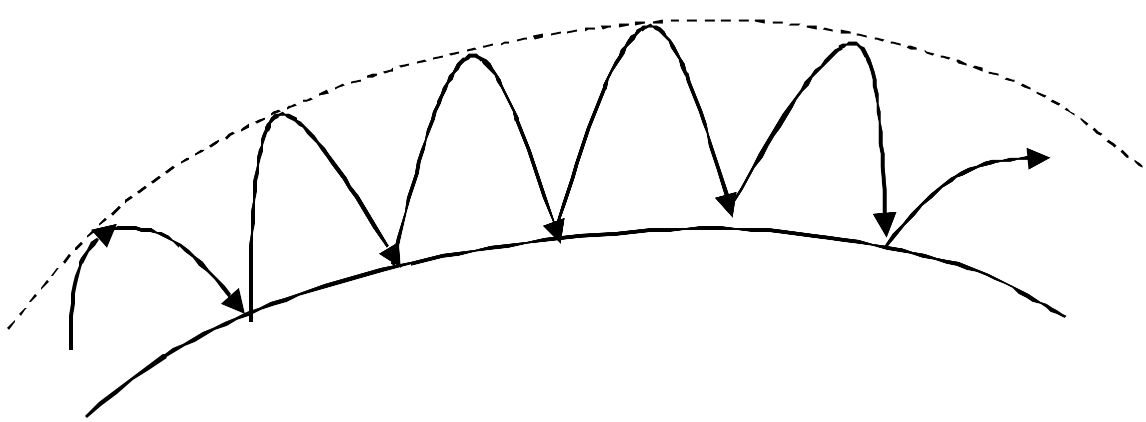
Antennas & Propagation

LECTURE NOTES
VOLUME V

ELECTROMAGNETIC WAVE PROPAGATION

by Professor David Jenn



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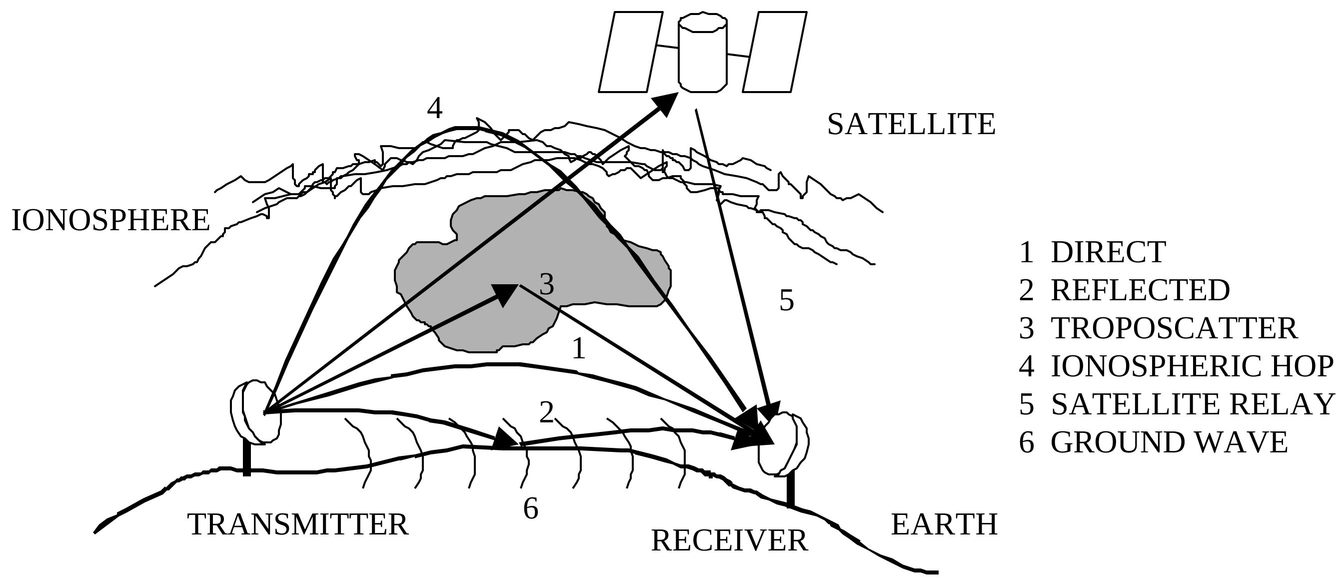
Propagation of Electromagnetic Waves

Radiating systems must operate in a complex changing environment that interacts with propagating electromagnetic waves. Commonly observed propagation effects are depicted below.



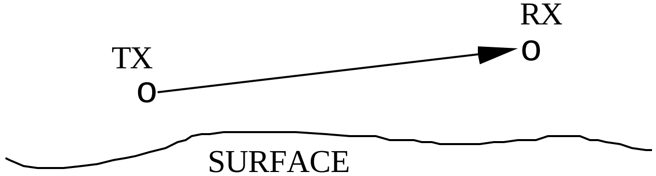
<u>Troposphere</u>: lower regions of the atmosphere (less than 10 km)
<u>Ionosphere</u>: upper regions of the atmosphere (50 km to 1000 km)

Effects on waves: reflection, refraction, diffraction, attenuation, scattering, and depolarization.

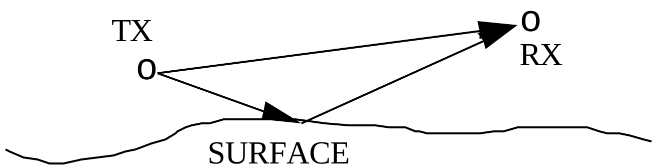
Survey of Propagation Mechanisms (1)

There are may propagation mechanisms by which signals can travel between the radar transmitter and receiver. Except for line-of-sight (LOS) paths, the mechanism's effectiveness is generally a strong function of the frequency and transmitter-receiver geometry.

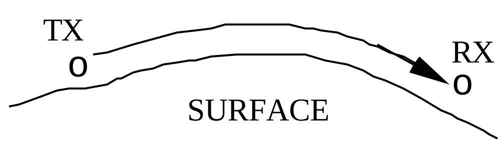
1. direct path or "line of sight" (most radars; SHF links from ground to satellites)



2. direct plus earth reflections or "multipath" (UHF broadcast; ground-to-air and air-to-air communications)

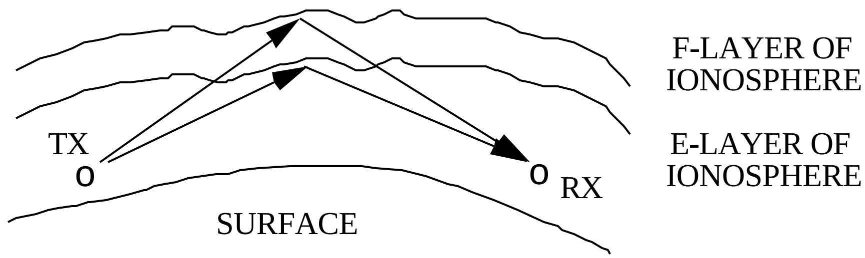


3. ground wave (AM broadcast; Loran C navigation at short ranges)

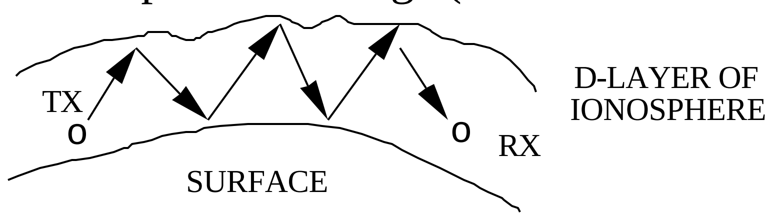


Survey of Propagation Mechanisms (2)

4. ionospheric hop (MF and HF broadcast and communications)



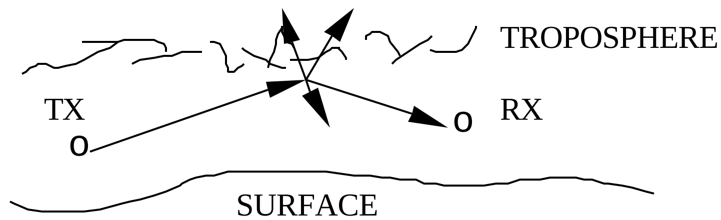
5. waveguide modes or "ionospheric ducting" (VLF and LF communications)



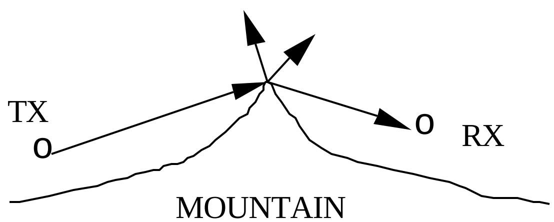
Note: The distinction between ionospheric hops and waveguide modes is based more on the mathematical models than on physical processes.

Survey of Propagation Mechanisms (3)

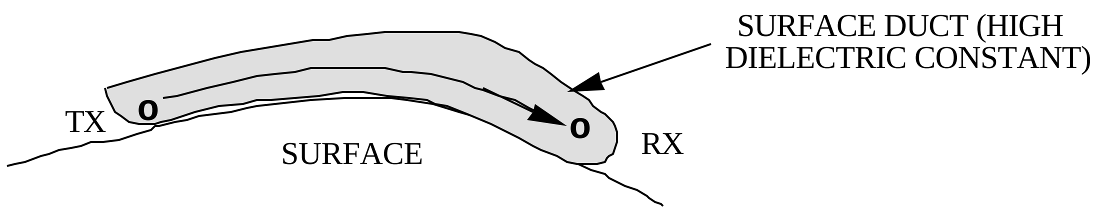
6. tropospheric paths or "troposcatter" (microwave links; over-the-horizon (OTH) radar and communications)



7. terrain diffraction

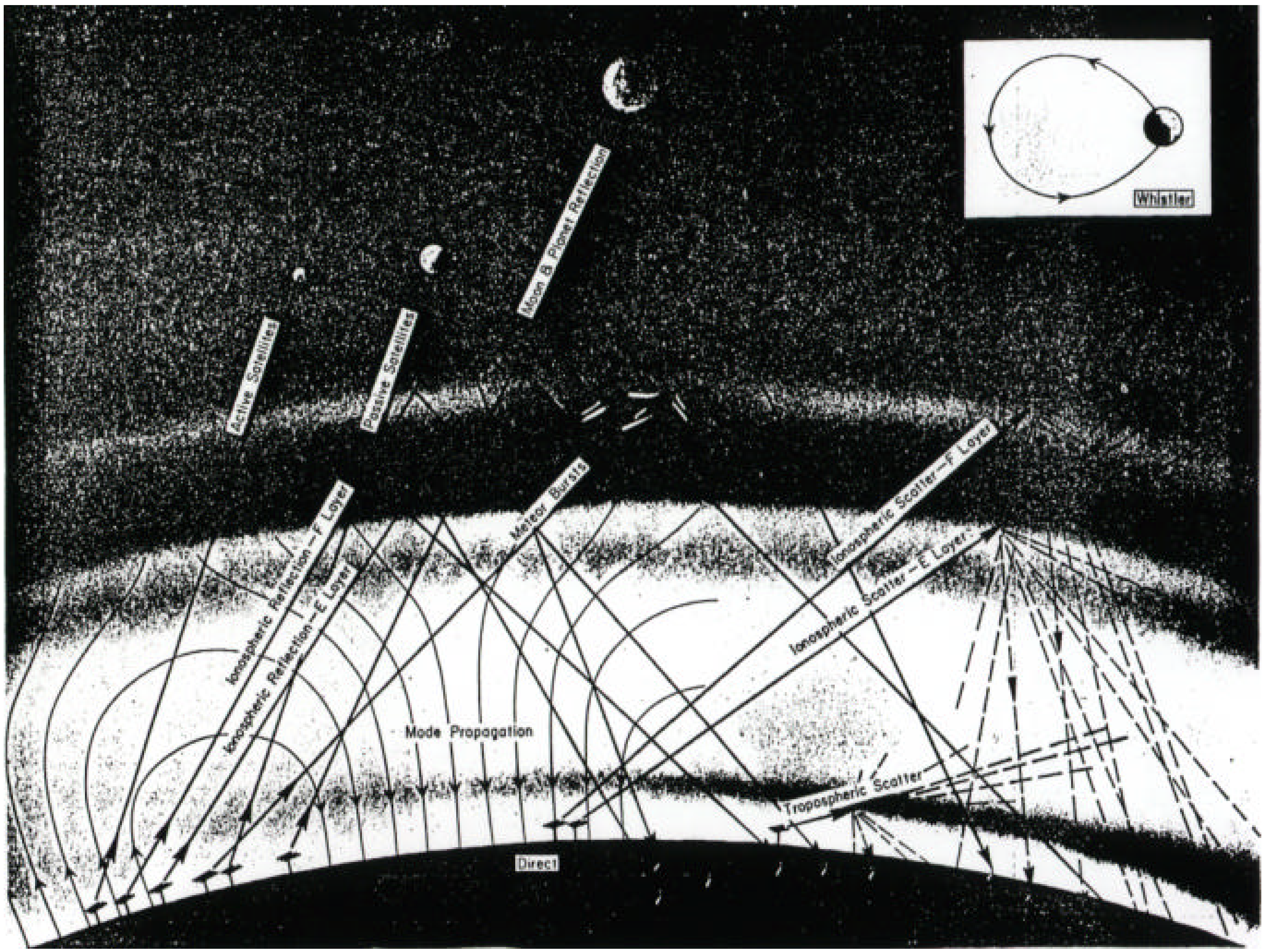


8. low altitude and surface ducts (radar frequencies)



9. Other less significant mechanisms: meteor scatter, whistlers

Illustration of Propagation Phenomena



(From Prof. C. A. Levis, Ohio State University)

Propagation Mechanisms by Frequency Bands

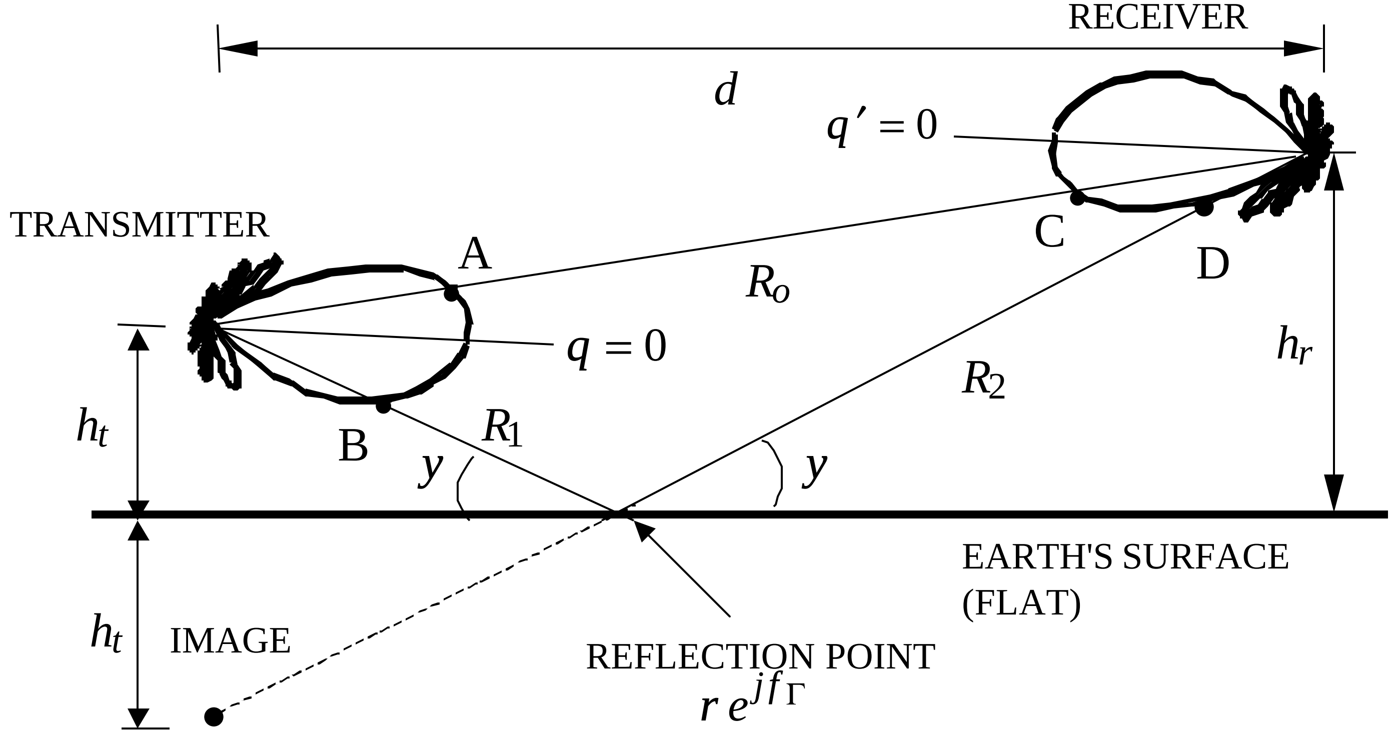
VLF and LF (10 to 200 kHz)	Waveguide mode between Earth and D-layer; ground wave at short distances
LF to MF (200 kHz to 2 MHz)	Transition between ground wave and mode predominance and sky wave (ionospheric hops). Sky wave especially pronounced at night.
HF (2 MHz to 30 MHz)	Ionospheric hops. Very long distance communications with low power and simple antennas. The "short wave" band.
VHF (30 MHz to 100 MHz)	With low power and small antennas, primarily for local use using direct or direct-plus-Earth-reflected propagation; ducting can greatly increase this range. With large antennas and high power, ionospheric scatter communications.
UHF (80 MHz to 500 MHz)	Direct: early-warning radars, aircraft-to satellite and satellite-to-satellite communications. Direct-plus-Earth-reflected: air-to-ground communications, local television. Tropospheric scattering: when large highly directional antennas and high power are used.
SHF (500 MHz to 10 GHz)	Direct: most radars, satellite communications. Tropospheric refraction and terrain diffraction become important in microwave links and in satellite communication, at low altitudes.

Applications of Propagation Phenomena

Direct	Most radars; SHF links from ground to satellites
Direct plus Earth reflections	UHF broadcast TV with high antennas; ground-to-air and air-to-ground communications
Ground wave	Local Standard Broadcast (AM), Loran C navigation at relatively short ranges
Tropospheric paths	Microwave links
Waveguide modes	VLF and LF systems for long-range communication and navigation (Earth and D-layer form the waveguide)
Ionospheric hops (E- and F-layers)	MF and HF broadcast communications (including most long-distance amateur communications)
Tropospheric scatter	UHF medium distance communications
Ionospheric scatter	Medium distance communications in the lower VHF portion of the band
Meteor scatter	VHF long distance low data rate communications

Multipath From a Flat Ground (1)

When both a transmitter and receiver are operating near the surface of the earth, multipath (multiple reflections) can cause fading of the signal. We examine a single reflection from the ground assuming a flat earth.



The reflected wave appears to originate from an image.

Multipath From a Flat Ground (2)

Multipath parameters:

1. Reflection coefficient, $\Gamma = r\, e^{j f_\Gamma}$. For low grazing angles, $y \approx 0$, the approximation $\Gamma \approx -1$ is valid for both horizontal and vertical polarizations.
2. Transmit antenna gain: $G_t(q_A)$ for the direct wave; $G_t(q_B)$ for the reflected wave.
3. Receive antenna gain: $G_r(q_C)$ for the direct wave; $G_r(q_D)$ for the reflected wave.
4. Path difference: $\Delta R = \underbrace{(R_1 + R_2)}_{\text{REFLECTED}} - \underbrace{R_o}_{\text{DIRECT}}$

Gain is proportional to the square of the electric field intensity. For example, if G_{to} is the gain of the transmit antenna in the direction of the maximum ($q = 0$), then

$$G_t(q) = G_{to} \left| E_{t_{\text{norm}}}(q) \right|^2 \equiv G_{to}\, f_t(q)^2$$

where $E_{t_{\text{norm}}}$ is the normalized electric field intensity. Similarly for the receive antenna with its maximum gain in the direction $q' = 0$

$$G_r(q') = G_{ro} \left| E_{r_{\text{norm}}}(q') \right|^2 \equiv G_{ro}\, f_r(q')^2$$

Multipath From a Flat Ground (3)

Total field at the receiver

$$\left|E_{\text{tot}}\right| = \underbrace{E_{\text{ref}}}_{\text{REFLECTED}} + \underbrace{E_{\text{dir}}}_{\text{DIRECT}}$$

$$= \left| f_t(q_A)\, f_r(q_C)\, \frac{e^{-jkR_o}}{4p\, R_o} \overbrace{\left[1 + \Gamma \frac{f_t(q_B)\, f_r(q_D)}{f_t(q_A)\, f_r(q_C)} e^{-jk\Delta R} \right]}^{\equiv F} \right|$$

The quantity in the square brackets is the <u>path-gain factor</u> (PGF) or <u>pattern-propagation</u> factor (PPF). It relates the total field at the receiver to that of free space and takes on values $0 \le F \le 2$.

- If $F = 0$ then the direct and reflected rays cancel (destructive interference)
- If $F = 2$ the two waves add (constructive interference)

Note that if the transmitter and receiver are at approximately the same heights, close to the ground, and the antennas are pointed at each other, then $d >> h_t, h_r$ and

$$G_t(q_A) \approx G_t(q_B)$$
$$G_r(q_C) \approx G_r(q_D)$$

Multipath From a Flat Ground (4)

An approximate expression for the path difference is obtained from a series expansion:

$$R_o = \sqrt{d^2 + (h_r - h_t)^2} \approx d + \frac{1}{2}\frac{(h_r - h_t)^2}{d}$$

$$R_1 + R_2 = \sqrt{d^2 + (h_t + h_r)^2} \approx d + \frac{1}{2}\frac{(h_t + h_r)^2}{d}$$

Therefore,

$$\Delta R \approx \frac{2h_r h_t}{d}$$

and

$$|F| = \left|1 - e^{-jk2h_r h_t/d}\right| = \left|e^{jkh_r h_t/d}\left(e^{-jkh_r h_t/d} - e^{jkh_r h_t/d}\right)\right| = 2\left|\sin\left(kh_r h_t/d\right)\right|$$

The received power depends on the square of the path gain factor

$$P_r \propto |F|^2 = 4\sin^2\left(\frac{kh_t h_r}{d}\right) \approx 4\left(\frac{kh_t h_r}{d}\right)^2$$

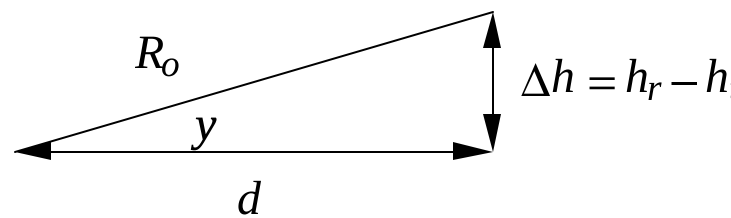
The last approximation is based on $h_r, h_t << d$ and $\Gamma \approx$ -1.

Multipath From a Flat Ground (5)

Two different forms of the argument are frequently used.

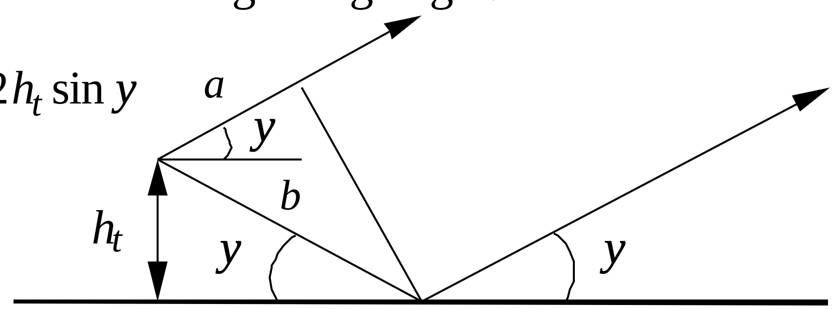
1. Assume that the transmitter is near the ground $h_t \approx 0$ and use its height as a reference. The elevation angle is y where

$$\tan y = \frac{h_r - h_t}{d} \equiv \frac{\Delta h}{d} \approx \frac{h_r}{d}$$



2. If the transmit antenna is very close to the ground, then the reflection point is very near to the transmitter and y is also the grazing angle:

$$\Delta R = b - a = 2h_t \sin y$$



If the antenna is pointed at the horizon (i.e., its maximum is parallel to the ground) then $y \approx q_A$.

Multipath From a Flat Ground (6)

Thus with the given restrictions the PPF can be expressed in terms of y

$$|F| = 2\sin\left(kh_t \tan y\right)$$

The PPF has minima at:

$$kh_t \tan y = np \quad (n = 0, 1, \ldots, \infty)$$

$$\frac{2p}{l} h_t \tan y = np$$

$$\tan y = n l / h_t$$

Maxima occur at:

$$kh_t \tan y = mp / 2 \quad (m = 1, 3, 5, \ldots, \infty)$$

$$\frac{2p}{l} h_t \tan y = \frac{2n+1}{2} p \quad (n = 0, 1, \ldots, \infty)$$

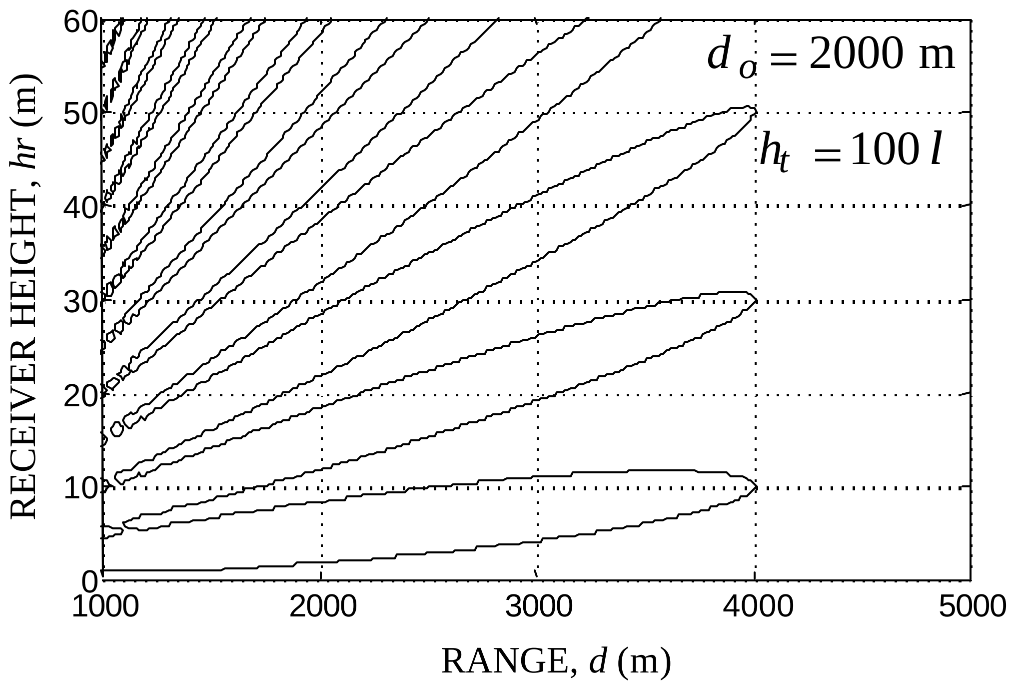
$$\tan y = \frac{(2n+1)\, l}{4h_t}$$

Plots $|F|$ are called a <u>coverage diagram</u>. The horizontal axis is usually distance and the vertical axis receiver height. (Note that because $d >> h_r$ the angle y is not directly measurable from the plot.)

Multipath From a Flat Ground (7)

<u>Coverage diagram</u>: Contour plots of $|F|$ in dB for variations in h_r and d normalized to a reference range d_o. Note that when $d = d_o$ then $E_{\text{tot}} = E_{\text{dir}}$.

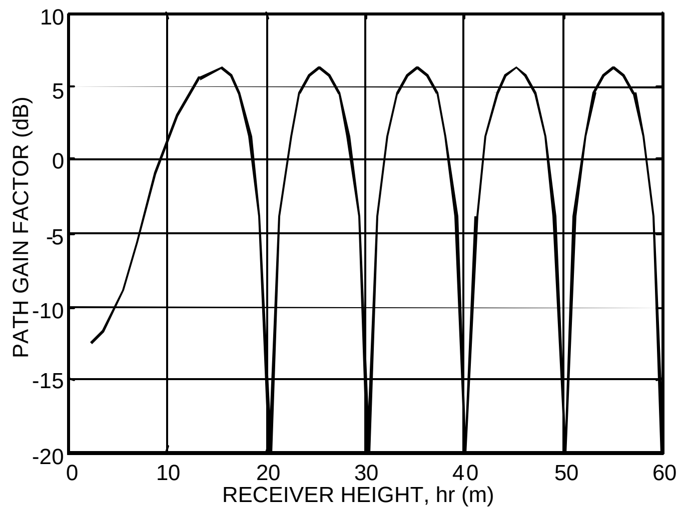
$$|F| = \left| 2\left(\frac{d_o}{d}\right)\sin\left(kh_t \tan\psi\right) \right|$$



Multipath From a Flat Ground (8)

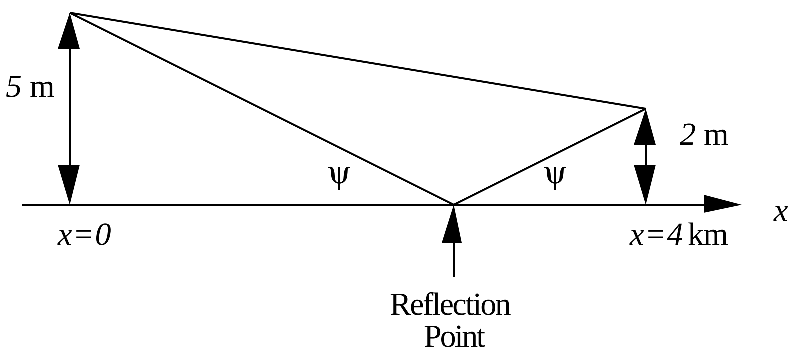
Another means of displaying the received field is a height-gain curve. It is a plot of $|F|$ in dB vs h_r at a fixed range.

- The constructive and destructive interference as a function of height can be identified.
- At low frequencies the periodicity of the curve at low heights can be destroyed by the ground wave.
- Usually there are many reflected wave paths between the transmitter and receiver, in which case the peaks and nulls are distorted.
- This technique is often used to determine the optimum tower height for a broadcast radio antenna.



Multipath Example

A radar antenna is mounted on a 5 m mast and tracks a point target at 4 km. The target is 2 m above the surface and the wavelength is 0.2 m. (a) Find the location of the reflection point on the x axis and the grazing angle y . (b) Write an expression for the one way path gain factor $|F|$ when a reflected wave is present. Assume a reflection coefficient of $\Gamma \approx -1$.



(a) Denote the location of the reflection point by x_r and use similar triangles

$$\tan y = \frac{5}{x_r} = \frac{2}{4000 - x_r}$$

$$x_r = 2.86 \text{ km}$$

$$y = \tan^{-1}(5/2860) = 0.1^\circ$$

(b) The restrictions on the heights and distance are satisfied for the following formula

$$|F| = 2\left|\sin\left(\frac{kh_t h_r}{d}\right)\right| = 2\left|\sin\left(\frac{2p\,(2)(5)}{(0.2)(4000}\right)\right|$$

$$= (2)(0.785) = 0.157$$

The received power varies as $|F|^2$, thus

$$10\log\left(|F|^2\right) = -16.1 \text{ dB}$$

The received power is 16.1 dB below the free space value

Field Intensity From the ERP

The product $P_t G_t$ is called the <u>effective radiated power</u> (ERP, or sometimes the <u>effective isotropic radiated power</u>, EIRP). We can relate the ERP to the electric field intensity as follows:

- The Poynting vector for a TEM wave:

$$\vec{W} = \Re\left\{\vec{E} \times \vec{H}^*\right\} = \frac{\left|\vec{E}_{\mathrm{dir}}\right|^2}{h_o}$$

- For the direct path:

$$\left|\vec{W}\right| = \frac{P_t G_t}{4pR_o^2}$$

- Equate the two expressions: (note that $h_o \approx 120p$)

$$\frac{\left|\vec{E}_{\mathrm{dir}}\right|^2}{h_o} = \frac{P_t G_t}{4pR_o^2} \quad \Rightarrow \quad \left|\vec{E}_{\mathrm{dir}}\right| = \frac{\sqrt{30 P_t G_t}}{d} \equiv \frac{E_o}{d}$$

where E_o is called the unattenuated field intensity at unit distance.

Wave Reflection at the Earth's Surface (1)

Fresnel reflection coefficients hold when:

1. the Earth's surface is locally flat in the vicinity of the reflection point
2. the surface is smooth (height of irregularities << l)

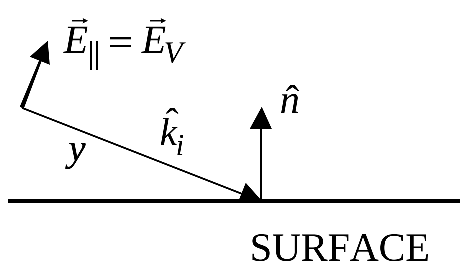
Traditional notation:

1. grazing angle, $y = 90^\circ - q_i$, and the grazing angle is usually very small ($y < 1^\circ$)
2. complex dielectric constant, $e_c = e_r e_o - j\dfrac{s}{w} = e_o\left(e_r - j\dfrac{s}{e_o w}\right) \equiv e_o \underbrace{(e_r - jc)}_{e_{rc}}$,

   where $c = \dfrac{s}{w e_o}$
3. horizontal and vertical polarization reference is used

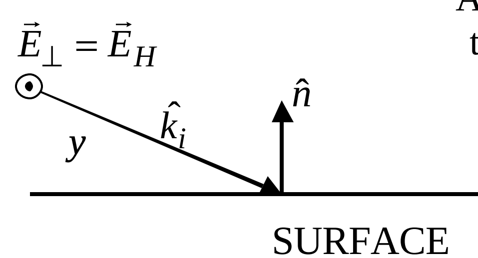
VERTICAL POL

Also called transverse magnetic (TM) pol



HORIZONTAL POL

Also called transverse electric (TE) pol



Wave Reflection at the Earth's Surface (2)

Reflection coefficients for horizontal and vertical polarizations:

$$-\Gamma_{\parallel} \equiv R_V = \frac{(e_r - jc)\sin y - \sqrt{(e_r - jc) - \cos^2 y}}{(e_r - jc)\sin y + \sqrt{(e_r - jc) - \cos^2 y}}$$

$$\Gamma_{\perp} \equiv R_H = \frac{\sin y - \sqrt{(e_r - jc) - \cos^2 y}}{\sin y + \sqrt{(e_r - jc) - \cos^2 y}}$$

For vertical polarization the phenomenon of total reflection can occur. This yields a surface guided wave called a <u>ground wave</u>. From Snell's law, assuming $m_r = 1$ for the Earth,

$$\sin q_i = \sin q_r = \sqrt{(e_r - jc)m_r}\,\sin q_t \quad \underset{m_r = 1}{\Rightarrow} \quad \sin q_t = \frac{\sin q_i}{\sqrt{e_r - jc}}$$

Let q_t be complex, $q_t = \frac{p}{2} + jq$, where q is real.

Wave Reflection at the Earth's Surface (3)

Using $q_t = \frac{p}{2} + jq$:

$$\sin q_t = \sin\left(\frac{p}{2} + jq\right) = \cos(jq) = \cosh q$$

$$\cos q_t = -j\sin(jq) = -j\sinh q$$

Snell's law becomes

$$\sin q_t = \cosh q = \frac{\sin q_i}{\sqrt{e_r - jc}}$$

$$\cos q_t = \sqrt{1 - \sin^2 q_t} = \sqrt{1 - \cosh^2 q} = \sinh q$$

Reflection coefficient for vertical polarization:

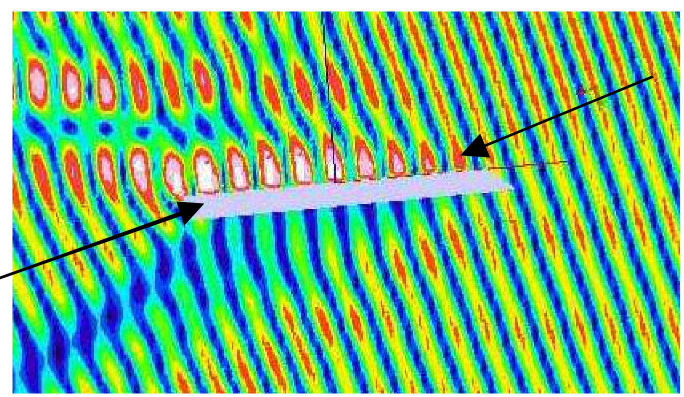
$$\Gamma_{\|} \equiv -R_V = \frac{jh \sinh q + h_o \cos q_i}{jh \sinh q - h_o \cos q_i}$$

where $h = \sqrt{\frac{m_o}{e_o(e_r - jc)}}$. Note that $|\Gamma_{\|}| = 1$ and therefore <u>all of the power flow is along the surface</u>. The wave decays exponentially with distance into the Earth.

Wave Reflection at the Earth’s Surface (4)

Example: surface wave propagating along a perfectly conducting plate

- $5l$ plate
- 15 degree grazing angle
- TM (vertical) polarization
- the total field is plotted (incident plus scattered)
- surface waves will follow curved surfaces if the radius of curvature >> l



Atmospheric Refraction (1)

Refraction by the lower atmosphere causes waves to be bent back towards the earth's surface. The ray trajectory is described by the equation: $n R_e \sin q = \mathrm{CONSTANT}$

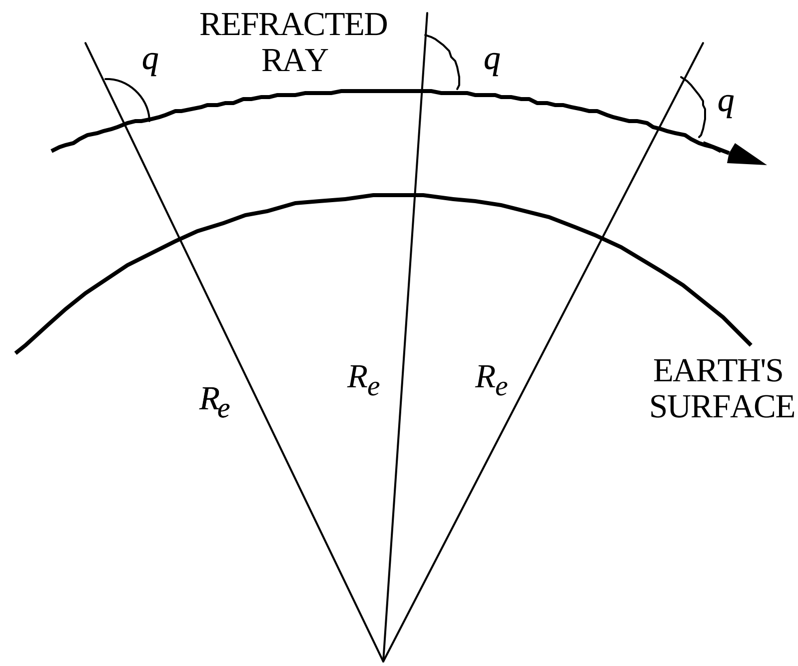
Two ways of expressing the index of refraction $n \left(= \sqrt{e_r}\right)$ in the troposphere:

1. $n = 1 + c\,r / r_{\mathrm{SL}}$ +HUMIDITY TERM

R_e = 6378 km = earth radius
$c \approx 0.00029 =$ Gladstone-Dale constant
$r,\ r_{SL}$ = mass densities at altitude and sea level

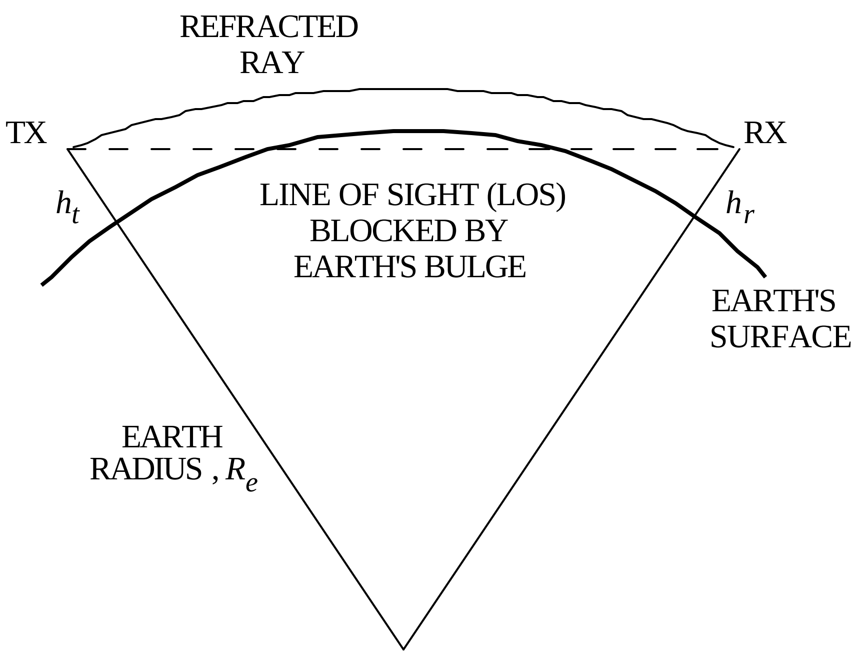
2. $n = \dfrac{77.6}{\mathrm{T}}\left(p + 4{,}810\,e/\mathrm{T}\right)10^{-6} - 1$

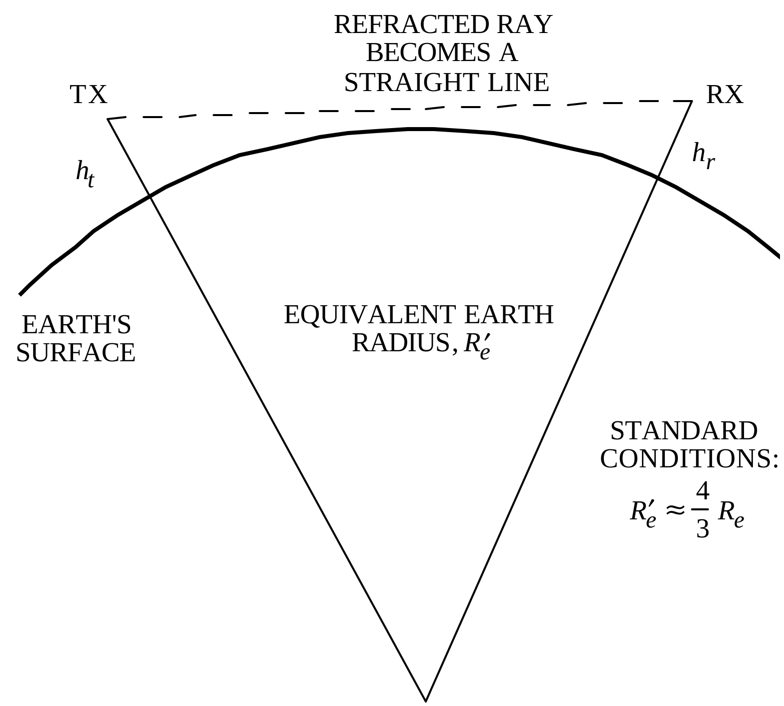
$p =$ air pressure (millibars)
T = temperature (K)
$e =$ partial pressure of water vapor (millibars)



Atmospheric Refraction (2)

Refraction of a wave can provide a significant level of transmission over the horizon. A bent refracted ray can be represented by a straight ray if an equivalent earth radius R_e' is used. For most atmospheric conditions $R_e' = 4R_e/3 = 8500$ km



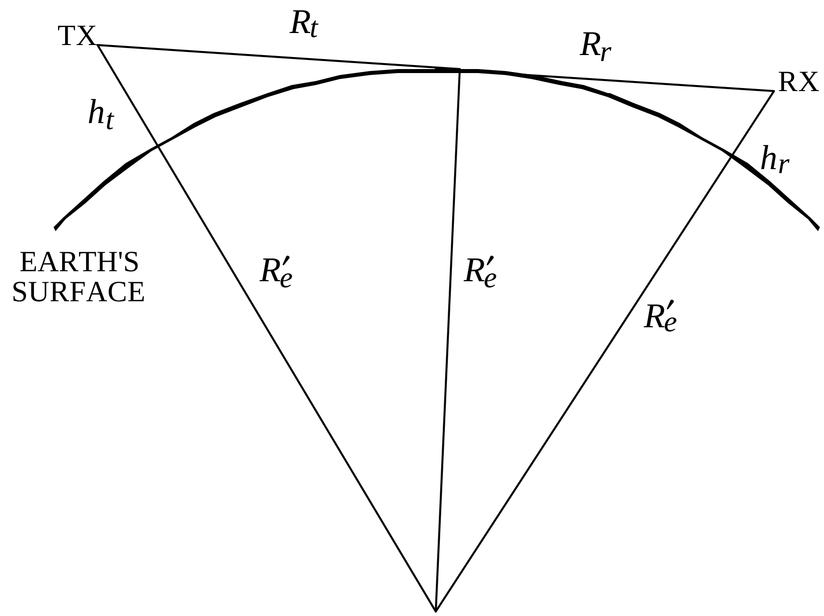


Atmospheric Refraction (3)

Distance from the transmit antenna to the horizon is $R_t = \sqrt{(R_e' + h_t)^2 - (R_e')^2}$ but $R_e' >> h_t$ so that $R_t \approx \sqrt{2R_e' h_t}$. Similarly $R_r \approx \sqrt{2R_e' h_r}$. The radar horizon is the sum

$$R_{RH} \approx \sqrt{2R_e' h_t} + \sqrt{2R_e' h_r}$$

Example: A missile is flying 15 m above the ocean towards a ground based radar. What is the approximate range that the missile can be detected assuming standard atmospheric conditions?

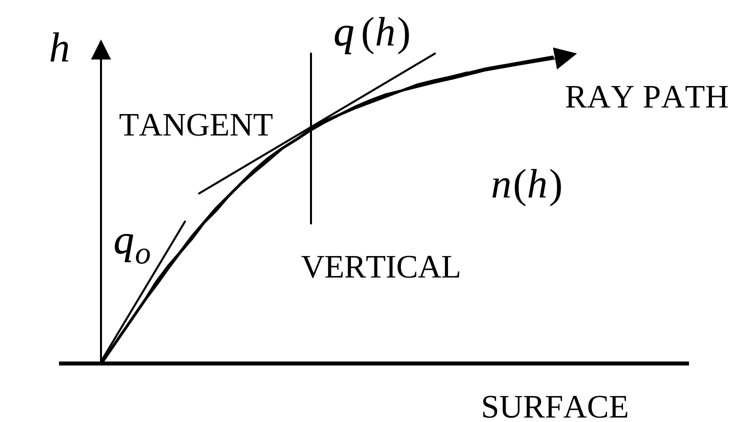


Using $h_t = 0$ and $h_r = 15$ gives a radar horizon of

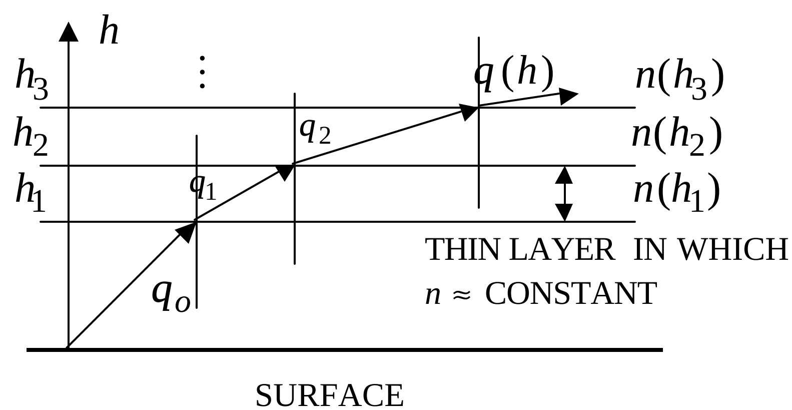
$$\begin{aligned} R_{RH} &\approx \sqrt{2R_e' h_r} \\ &\approx \sqrt{(2)(8500 \times 10^3)(15)} \\ &\approx 16\,\text{km} \end{aligned}$$

Atmospheric Refraction (4)

Derivation of the equivalent Earth radius



Break up the atmosphere into thin horizontal layers. Snell's law must hold at the boundary between each layer, $\sqrt{e(h)}\sin[q(h)] = \sqrt{e_o}\sin q_o$



Atmospheric Refraction (5)

In terms of the Earth radius,

$$\underbrace{R_e \sqrt{e_o}\sin q_o}_{\substack{\text{AT THE}\\ \text{SURFACE}}} = \underbrace{(R_e + h)\sqrt{e(R)}\sin[q(R)]}_{\substack{\text{AT RADIUS}\\ R=R_e+h}}$$

Using the grazing angle, and assuming that $e(h)$ varies linearly with h

$$R_e \sqrt{e_o}\cos y_o = (R_e + h)\left\{\sqrt{e_o} + h\frac{d}{dh}\sqrt{e(h)}\right\}\cos[y(h)]$$

Expand and rearrange

$$R_e \sqrt{e_o}\{\cos y_o - \cos[y(h)]\} = \left\{\sqrt{e_o} + R_e\frac{d}{dh}\sqrt{e(h)}\right\}h\cos[y(h)] + h^2\frac{d}{dh}\sqrt{e(h)}\cos[y(h)]$$

If $h << R_e$ then the last term can be dropped, and since y is small, $\cos y \approx 1 + y^2/2$

$$[y(h)]^2 \approx y_o^2 + \frac{2h}{R_e}\left[1 + \frac{R_e}{\sqrt{e_o}}\frac{d}{dh}\sqrt{e(h)}\right]$$

The second term is due to the inhomogenity of the index of refraction with altitude.

Atmospheric Refraction (6)

Define a constant k such that

$$[y(h)]^2 \approx y_o^2 + \frac{2h}{kR_e} = y_o^2 + \frac{2h}{R_e'} \qquad \text{where} \qquad k = \left[1 + \frac{R_e}{\sqrt{e_o}} \frac{d}{dh}\sqrt{e(h)}\right]^{-1}$$

$R_e' = kR_e$ is the effective (equivalent) Earth radius. If R_e' is used as the Earth radius then rays can be drawn as straight lines. This is the radius that would produce the same geometrical relationship between the source of the ray and the receiver near the Earth's surface, assuming a constant index of refraction. The restrictions on the model are:

1. Ray paths are nearly horizontal
2. $\sqrt{e(h)}$ versus h is linear over the range of heights considered

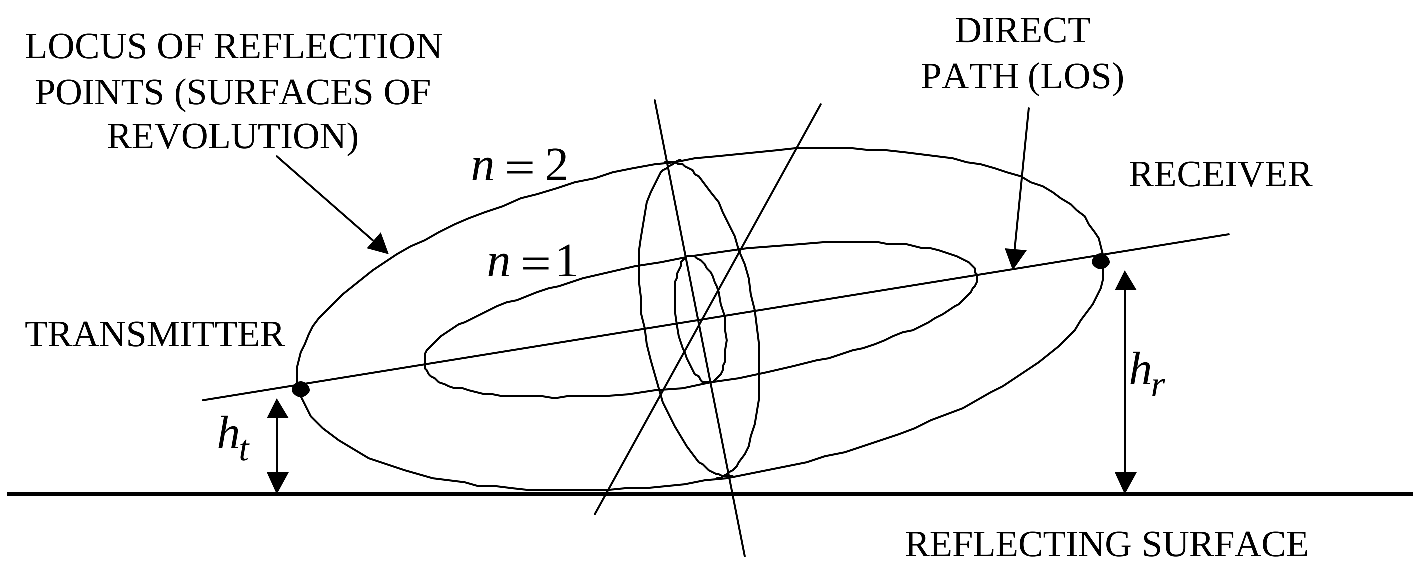
Under standard (normal) atmospheric conditions, $k \approx 4/3$. That is, the radius of the Earth is approximately $R_e' = \left(\frac{4}{3}\right)6378\,\text{km} = 8500\,\text{km}$. This is commonly referred to as "the four-thirds Earth approximation."

Fresnel Zones (1)

For the direct path phase to differ from the reflected path phase by an integer multiple of 180° the paths must differ by integer multiples of $l/2$

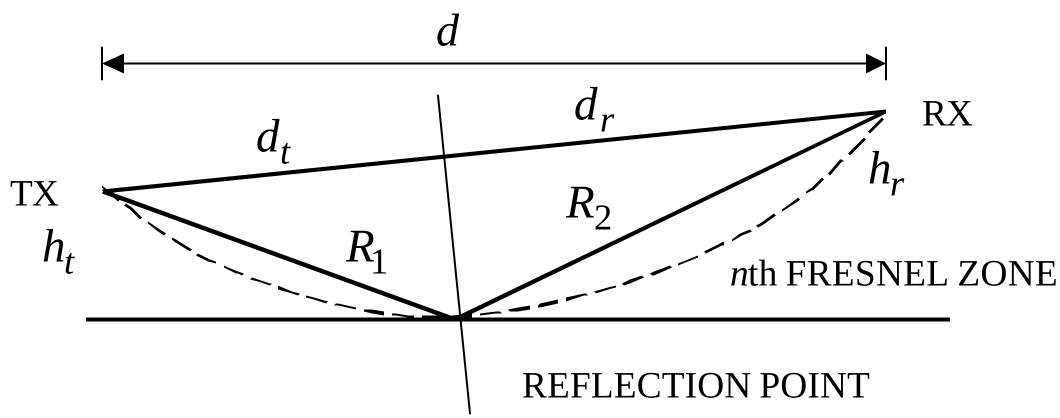
$$\Delta R = n l/2 \qquad (n = 0, 1, \ldots)$$

The collection of points at which reflection would produce an excess path length of $n l/2$ is called the n^{th} Fresnel zone. In three dimensions the surfaces are ellipsoids centered on the direct path between the transmitter and receiver



Fresnel Zones (2)

A slice of the vertical plane gives the following geometry



For the reflection coefficient $\Gamma = r\, e^{jp} = -r$:

- If <u>n is even</u> the two paths are out of phase and the received signal is a <u>minimum</u>
- If <u>n is odd</u> the two paths are in phase and the received signal is a <u>maximum</u>

Because the LOS is nearly horizontal $R_o \approx d$ and therefore $R_o = d_t + d_r \approx d$. For the nth Fresnel zone $R_1 + R_2 = d + n l / 2$.

Fresnel Zones (3)

The radius of the nth Fresnel zone is

$$F_n = \sqrt{\frac{n\, l\, d_t d_r}{d}}$$

or, if the distances are in miles, then

$$F_n = 72.1\sqrt{\frac{n d_t d_r}{f_{\mathrm{GHz}} d}} \text{ (feet)}$$

Transmission path design: the objective is to find transmitter and receiver locations and heights that give signal maxima. In general:

1. reflection points should not lie on even Fresnel zones
2. the LOS should clear all obstacles by $0.6F_1$, which essentially gives free space transmission

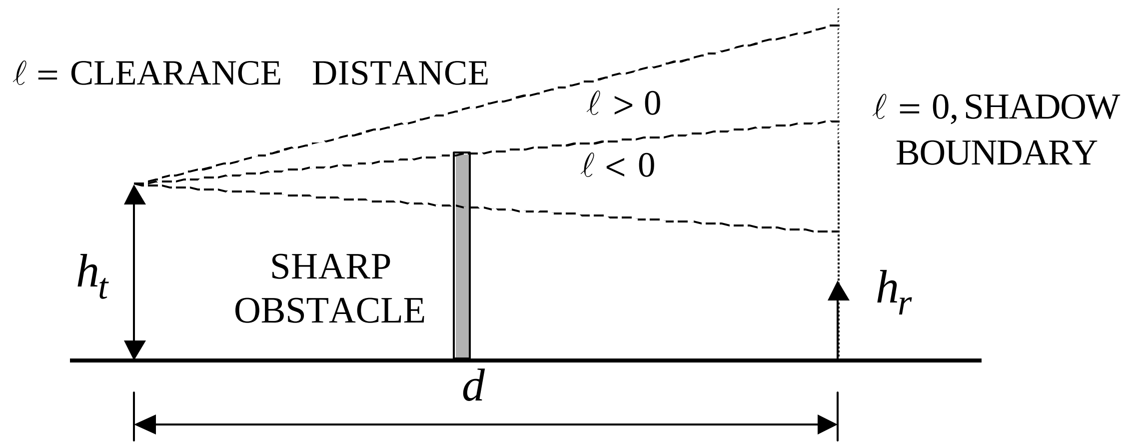
The significance of $0.6F_1$ is illustrated by examining two canonical problems:

(1) knife edge diffraction and
(2) smooth sphere diffraction.

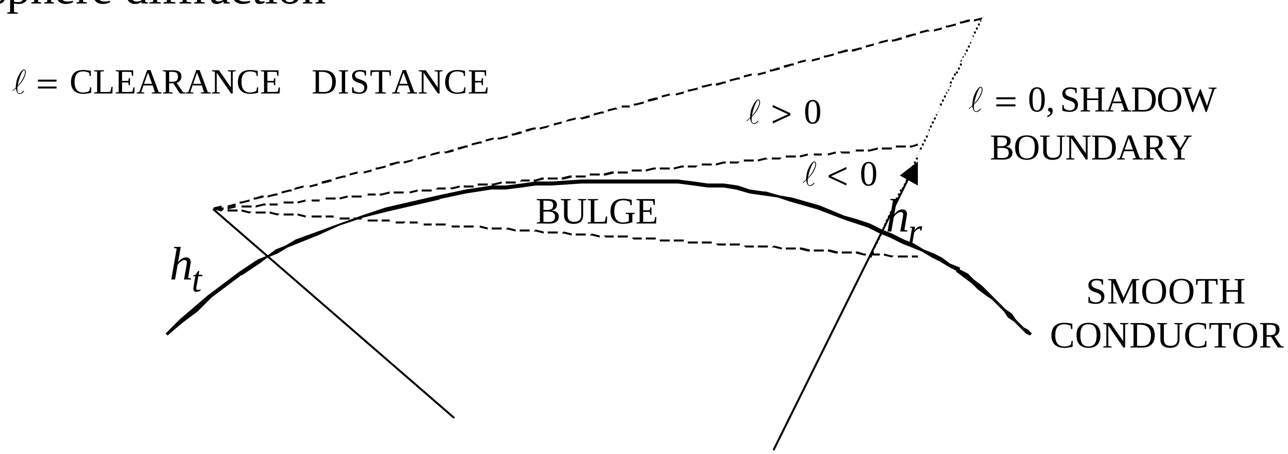
Conversions: 0.0254 m = 1 in; 12 in = 1 ft; 3.3 ft = 1 m; 5280 ft = 1 mi; 1 km = 0.62 mi

Diffraction (1)

Knife edge diffraction

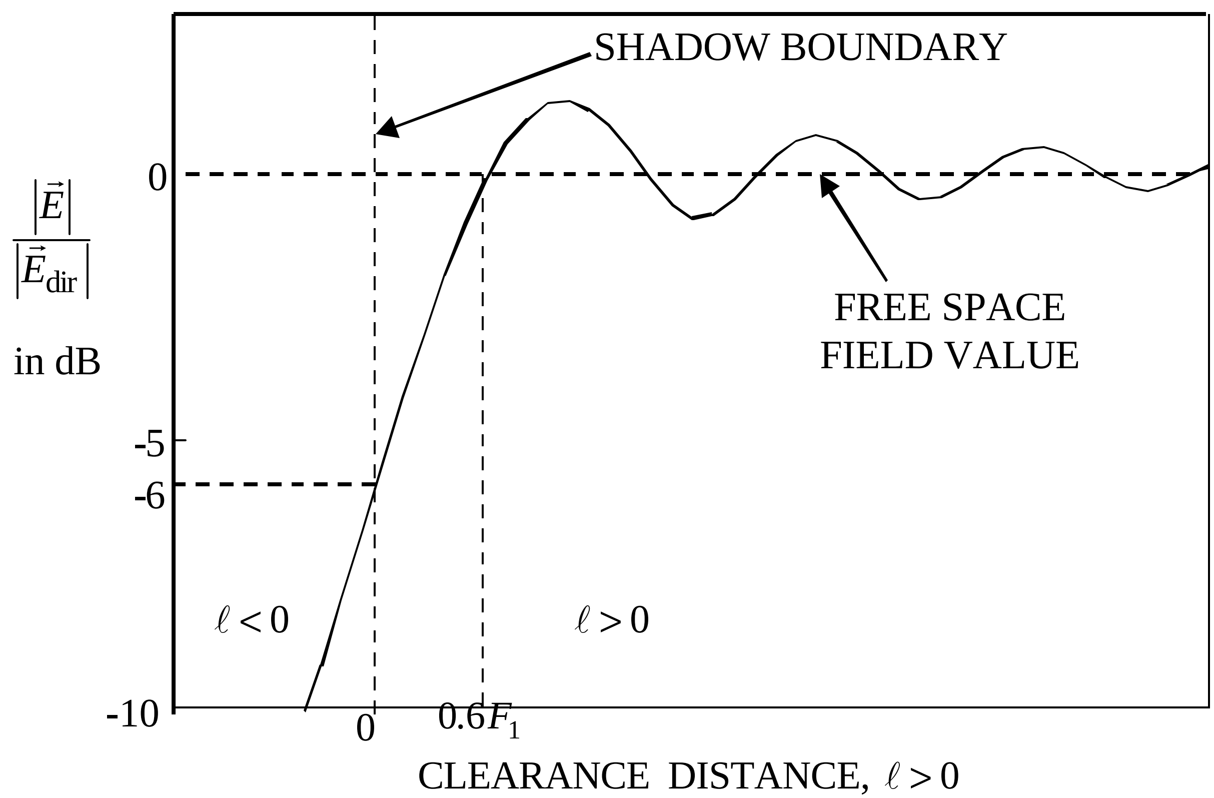


Smooth sphere diffraction



Diffraction (2)

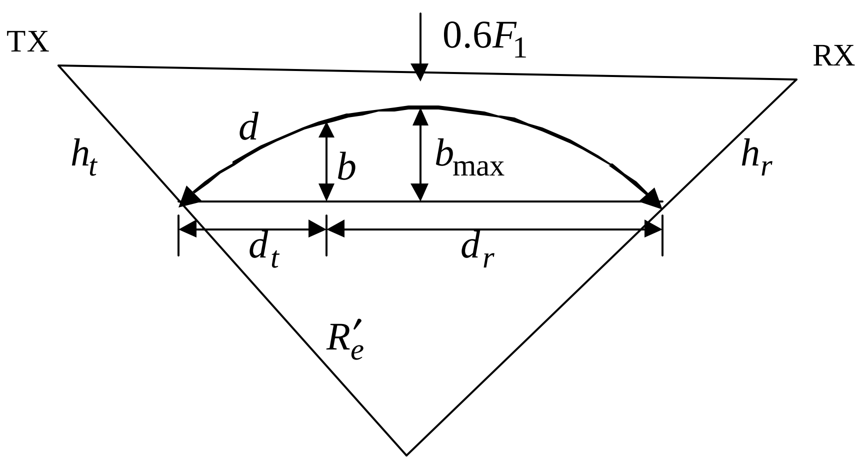
A plot of $\left|\frac{E_{\text{tot}}}{E_{\text{dir}}}\right|$ shows that at $0.6F_1$ the free space (direct path) value is obtained.



Path Clearance Example

Consider a 30 mile point-to-point communication link over the ocean. The frequency of operation is 5 GHz and the antennas are at the same height. Find the lowest height that provides the same field strength as in free space. Assume standard atmospheric conditions.

The geometry is shown below (distorted scale). The <u>bulge factor</u> (in feet) is given approximately by $b = \dfrac{d_t d_r}{1.5k}$, where d_t and d_r are in miles.



The maximum bulge occurs at the midpoint.

$$d \approx d_t + d_r$$

$$b_{\max} = \frac{(15)(15)}{(1.5)(4/3)} = 112.5\ \text{ft}$$

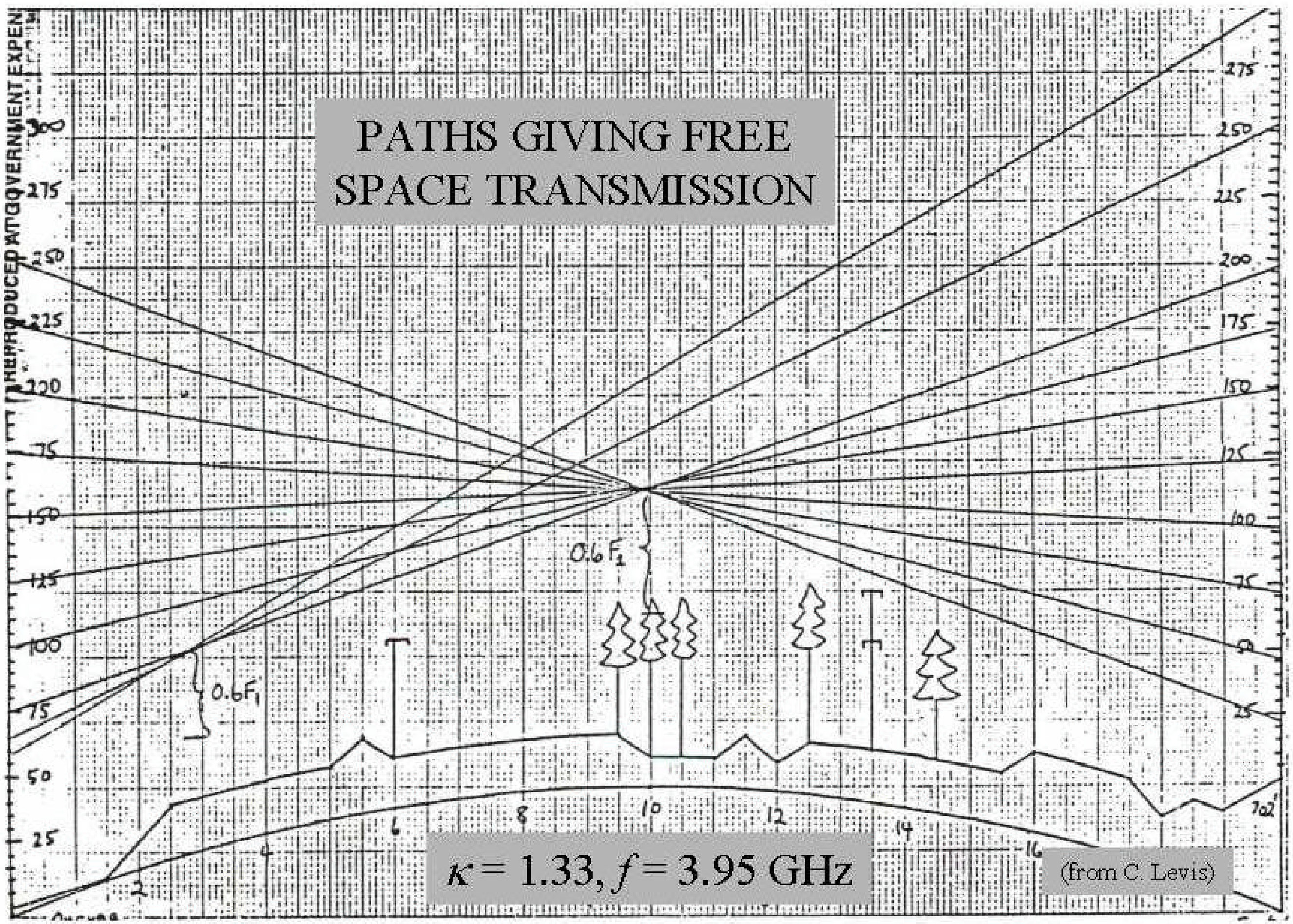
$$F_n = 72.1\sqrt{\frac{n d_t d_r}{f_{\text{GHz}} d}}\ \text{ft}$$

$$0.6F_1 = 53\ \text{ft}$$

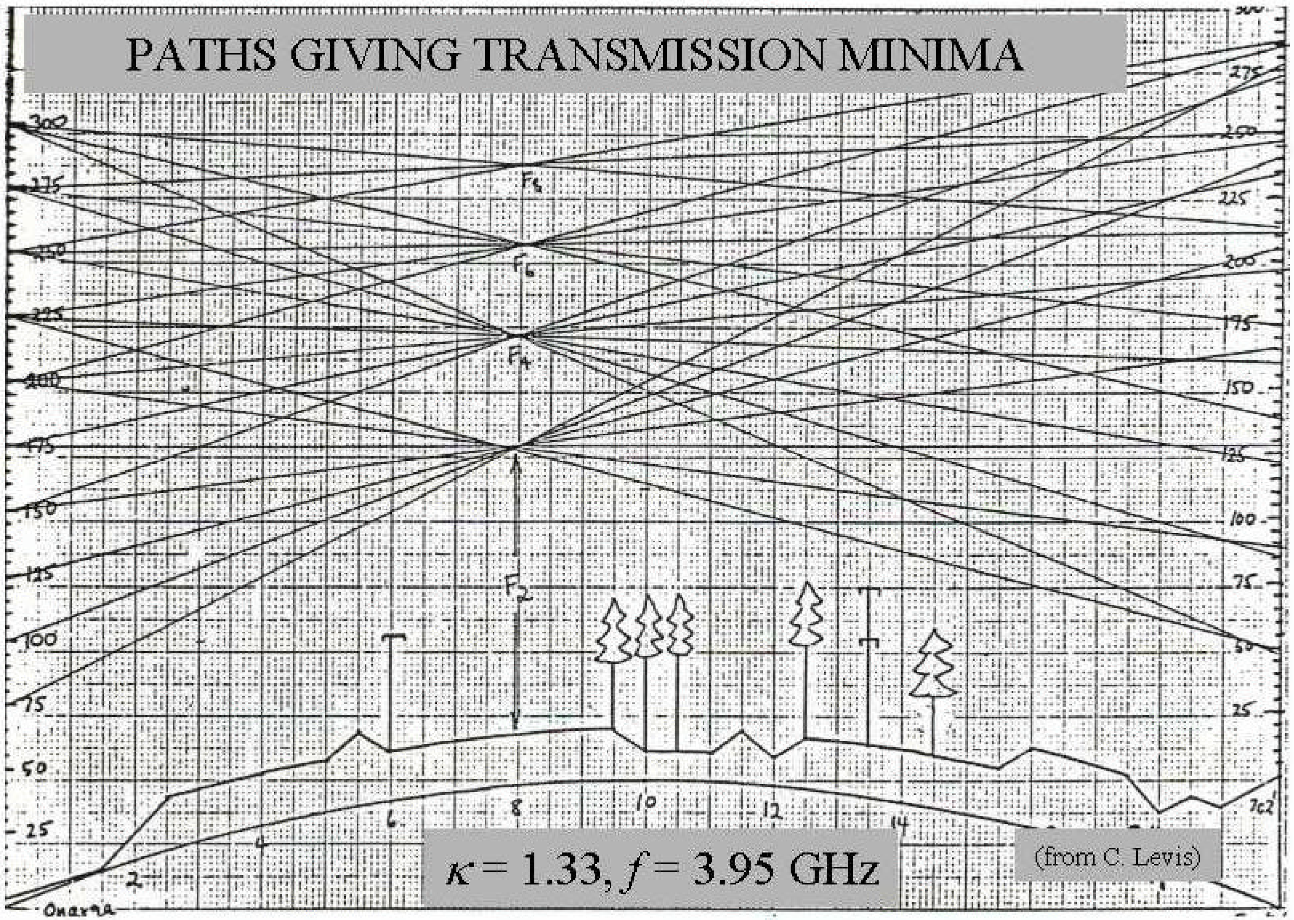
Compute the minimum antenna height:

$$h = b_{\max} + 0.6F_1$$
$$= 112.5 + 53 = 165\ \text{ft}$$

Example of Link Design (1)



Example of Link Design (2)



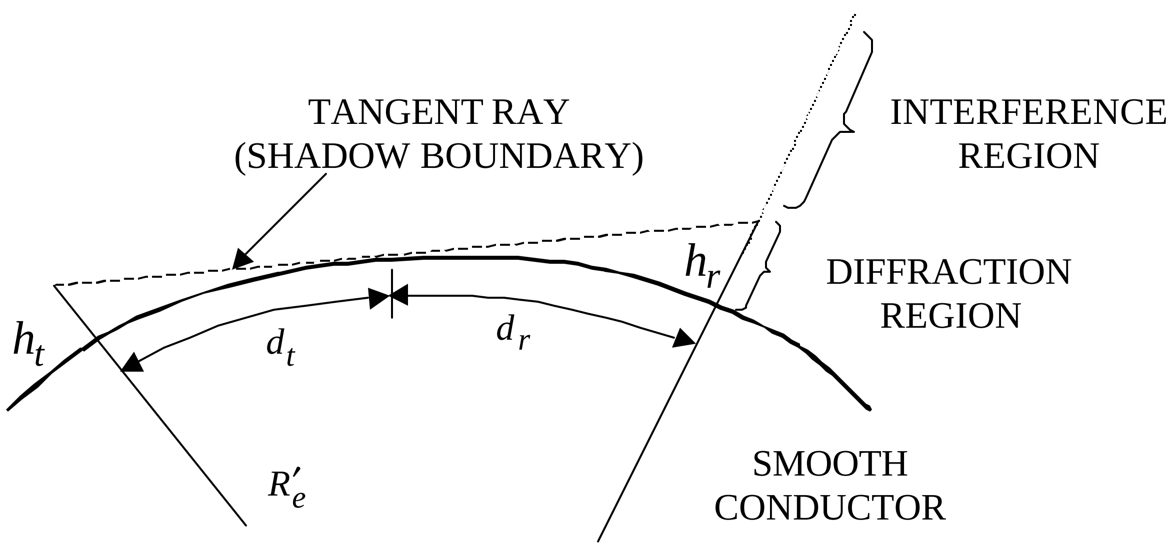
Antennas Over a Spherical Earth

When the transmitter to receiver distance becomes too large the flat Earth approximation is no longer accurate. The curvature of the surface causes:

1. divergence of the power in the reflected wave in the interference region
2. diffracted wave in the shadow region (note that this is not the same as a ground wave)

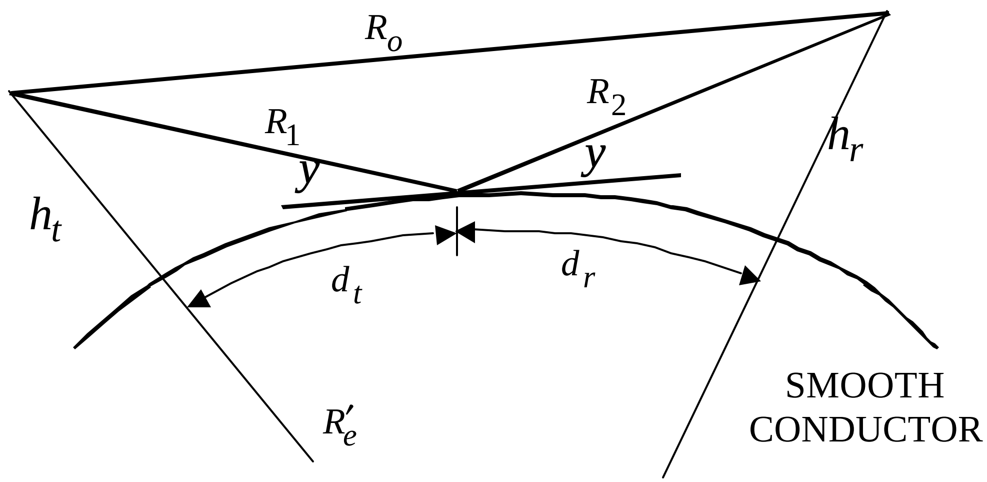
The distance to the horizon is $d_t = R_{RH} \approx \sqrt{2R'_e h_t}$ or, if h_t is in feet, $d_t \approx \sqrt{2h_t}$ miles. The maximum LOS distance between the transmit and receive antennas is

$$d_{\max} = d_t + d_r \approx \sqrt{2h_t} + \sqrt{2h_r} \quad \text{(miles)}$$



Interference Region Formulas (1)

Interference region formulas



The path-gain factor is given by

$$|F| = \left|1 + r\, e^{j f_\Gamma} e^{-jk\Delta R} \sqrt{D}\right|$$

where D is the divergence factor (power) and $\Delta R = R_1 + R_2 - R_o$.

Interference Region Formulas (2)

Approximate formulas[1] for the interference region:

$$|F| = \left\{ \left(1 + |\Gamma|\sqrt{D}\right)^2 - 4|\Gamma|\sqrt{D}\sin^2\left[\frac{f_\Gamma - k\Delta R}{2}\right] \right\}^{\frac{1}{2}}$$

where

$$\Delta R = \frac{2h_1h_2}{d} J(S,T), \quad \tan y = \frac{h_1 + h_2}{d} K(S,T), \quad D = \left[1 + \frac{4S_1S_2^2T}{S(1-S_2^2)(1+T)}\right]^{-1} \text{ (power)}$$

$$S_1 = \frac{d_1}{\sqrt{2R_e'h_1}}, \quad S_2 = \frac{d_2}{\sqrt{2R_e'h_2}} \quad \text{where } h_1 \text{ is the smallest of either } h_t \text{ or } h_r$$

$$S = \frac{d}{\sqrt{2R_e'h_1} + \sqrt{2R_e'h_2}} = \frac{S_1T + S_2}{1+T}, \quad T = \sqrt{h_1/h_2} \quad (< 1 \text{ since } h_1 < h_2)$$

$$J(S,T) = (1 - S_1^2)(1 - S_2^2), \text{ and } \quad K(S,T) = \frac{(1 - S_1^2) + T^2(1 - S_2^2)}{1 + T^2}$$

[1]D. E. Kerr, *Propagation of Short Radio Waves*, Radiation Laboratory Series, McGraw-Hill, 1951 (the formulas have been reprinted in many other books including R. E. Collin, *Antennas and Radiowave Propagation*, McGraw-Hill, 1985).

Interference Region Formulas (3)

The distances can be computed from $d = d_1 + d_2$ and

$$d_1 = \frac{d}{2} + p\cos\left(\frac{\Phi + p}{3}\right),\ \ \Phi = \cos^{-1}\left(\frac{2R_e'(h_1 - h_2)d}{p^3}\right),$$

and

$$p = \frac{2}{\sqrt{3}}\left[R_e'(h_1 + h_2) + \frac{d^2}{4}\right]^{1/2}$$

Another form for the phase difference is

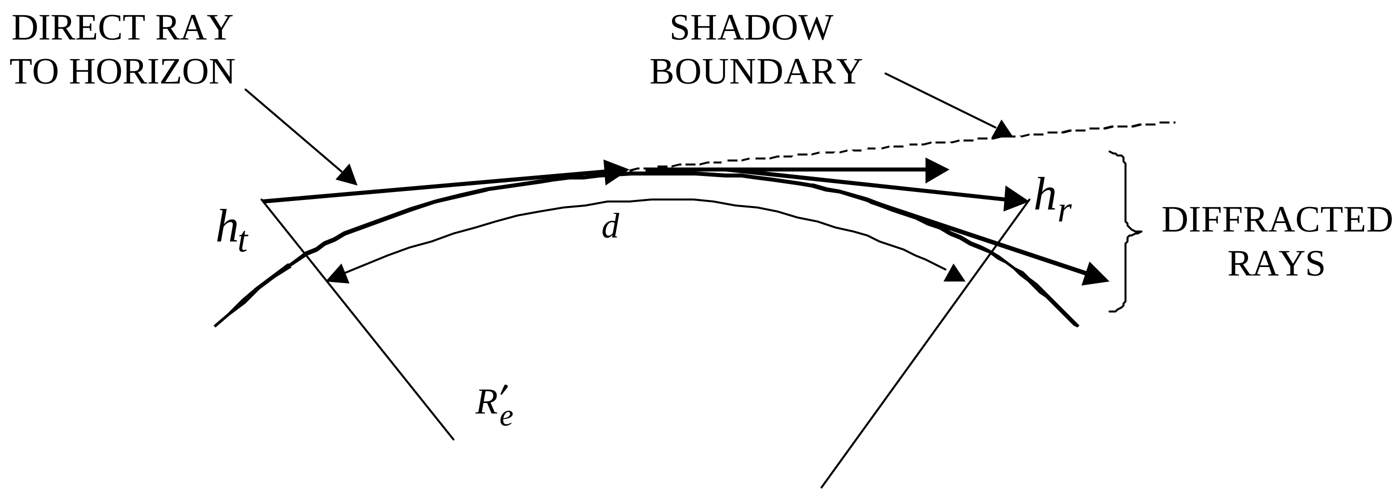
$$k\Delta R = \frac{2kh_1h_2}{d}(1 - S_1^2)(1 - S_2^2) = nzp$$

where

$$n = \frac{4h_1^{3/2}}{l\sqrt{2R_e'}} = \frac{h_1^{3/2}}{1030\,l},\quad z = \frac{h_2/h_1}{d/d_{RH}}(1 - S_1^2)(1 - S_2^2),$$

and $d_{RH} = \sqrt{2R_e'h_1}$ (distance to the radio horizon).

Diffraction Region Formulas (1)



Approximate formulas for the diffraction region (frequencies > 100 MHz):

$$F = V_1(X)U_1(Z_1)U_1(Z_2)$$

where U_1 is available from tables or curves, $Z_i = h_i / H \quad (i = 1,2)$, $X = d / L$, and

$$V_1(X) = 2\sqrt{pX}\,e^{-2.02X}, \; L = 2\left(\frac{(R'_e)^2}{4k}\right)^{\frac{1}{3}} = 28.41\,l^{1/3}(\text{km}), \; H = \left(\frac{R'_e}{2k^2}\right)^{\frac{1}{3}} = 47.55\,l^{2/3}(\text{m})$$

Diffraction Region Formulas (2)

A plot of $U_1(Z)$

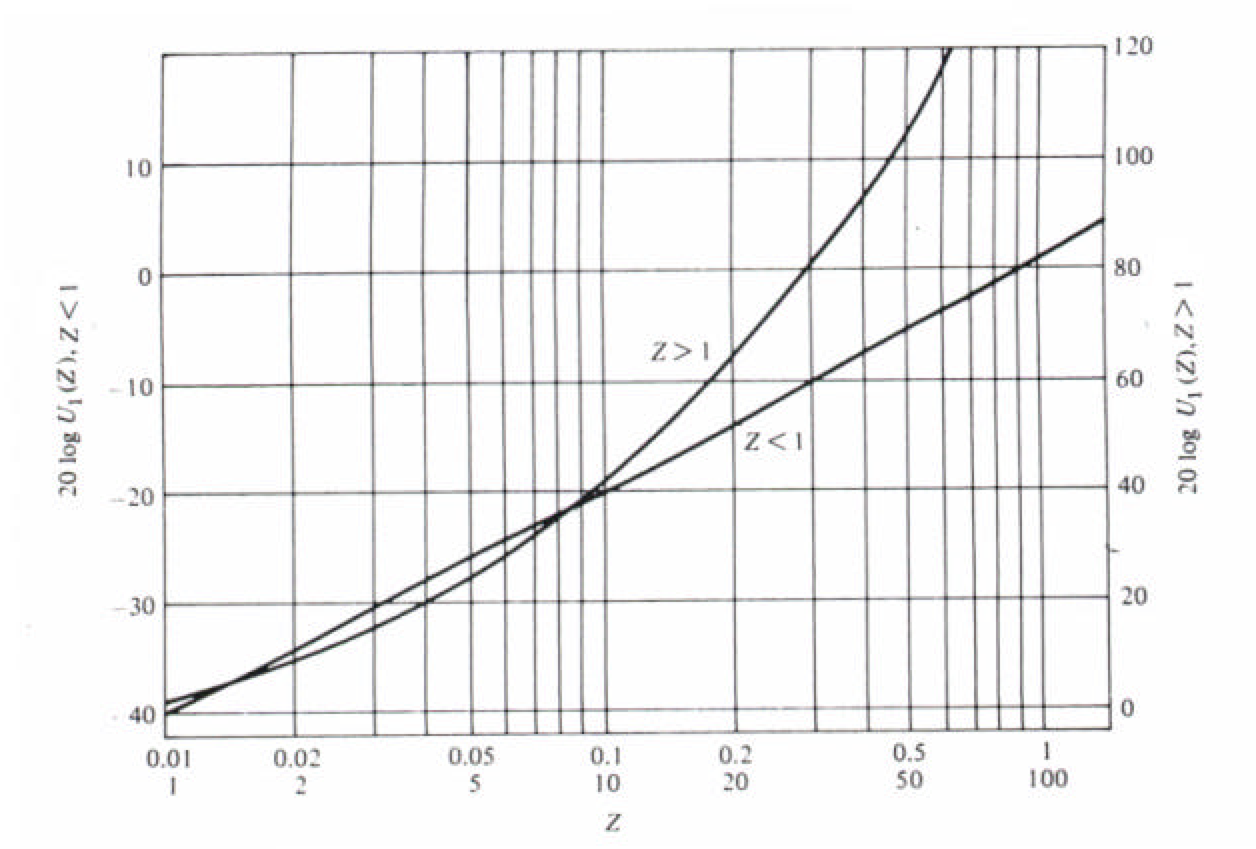
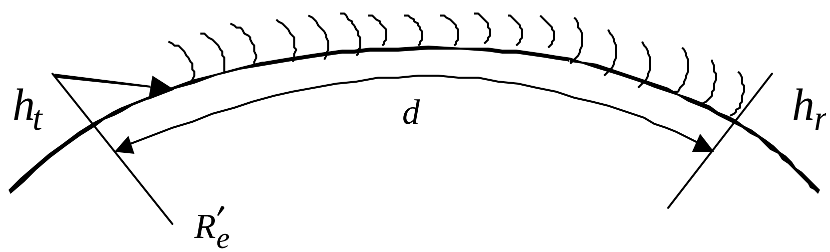


Fig. 6.29 in R. E. Collin, *Antennas and Radiowave Propagation*, McGraw-Hill, 1985 (axis labels corrected)

Surface Waves (1)

At low frequencies (1 kHz to about 3 MHz) the interface between air and the ground acts like an efficient waveguide at low frequencies for vertical polarization. Collectively the space wave (direct and Earth reflected) and surface wave are called the ground wave.



The power density at the receiver is the free space value times an attenuation factor

$$P_r = P_{\mathrm{dir}} \left| 2 A_s \right|^2$$

where the factor of 2 is by convention. Most estimates of A_s are based calculations for a surface wave along a flat interface. Approximations for a flat surface are good for $d \le 50/(f_{\mathrm{MHz}})^{1/3}$ miles. Beyond this distance the received signal attenuates more quickly.

Surface Waves (2)

Define a two parameters: $$p = \frac{kd}{2\sqrt{e_r^2 + (s\ / we_o)^2}} \quad \text{(numerical distance)}$$

$$b = \tan^{-1}\left(\frac{e_r e_o w}{s}\right)$$

A convenient formula is $s\ / we_o = \dfrac{1.8\times 10^4 s}{f_{\text{MHz}}}$. The attenuation factor for the ground wave is approximately $|A_s| = \dfrac{2+0.3p}{2+p+0.6p^2} - \sqrt{p/2}\, e^{-0.6p} \sin b \quad (b \le 90^\circ)$

Example: A CB link operates at 27 MHz with low gain antennas near the ground. Find the received power at the maximum flat Earth distance. The following parameters hold: $P_t = 5$ W; $G_t = G_r = 1$; $e_r = 12$ and $s = 5\times 10^{-3}$ S/m. The maximum flat Earth range is $d_{\text{max}} = 50/(27)^{1/3} = 16.5$ miles.

$$p = \frac{pd\ /\ l}{\sqrt{12^2 + (90/27)^2}} = 0.25\, d\ /\ l = 0.0225\left(\frac{16.5}{0.62}\right)(1000) \approx 601 \quad \rightarrow \quad \frac{d}{l} = 4p$$

Surface Waves (3)

Check b to see if formula applies (otherwise use the chart on the next page)

$$b = \tan^{-1}\left(\frac{(12)(8.85\times 10^{-12})(2p)(27\times 10^{6})}{5\times 10^{-3}}\right) = 74.5^\circ$$

Attenuation constant

$$|A_s| = \frac{2+0.3p}{2+p+0.6p^2} - \sqrt{p/2}\, e^{-0.6p} \sin b \approx 8.33\times 10^{-4}$$

The received power for the ground wave is

$$P_r = P_{\text{dir}}|2A_s|^2 = \frac{P_t G_t A_{er}}{4p\, d^2}|2A_s|^2 = \frac{P_t(1)\left(l^2/4p\right)}{4p\, d^2}|2A_s|^2$$

$$= \frac{(5)(8.33\times 10^{-4})^2}{(4p)(4)^2(601)^2} = 1.52\times 10^{-14}\ \text{W}$$

Surface Waves (4)

FLAT EARTH

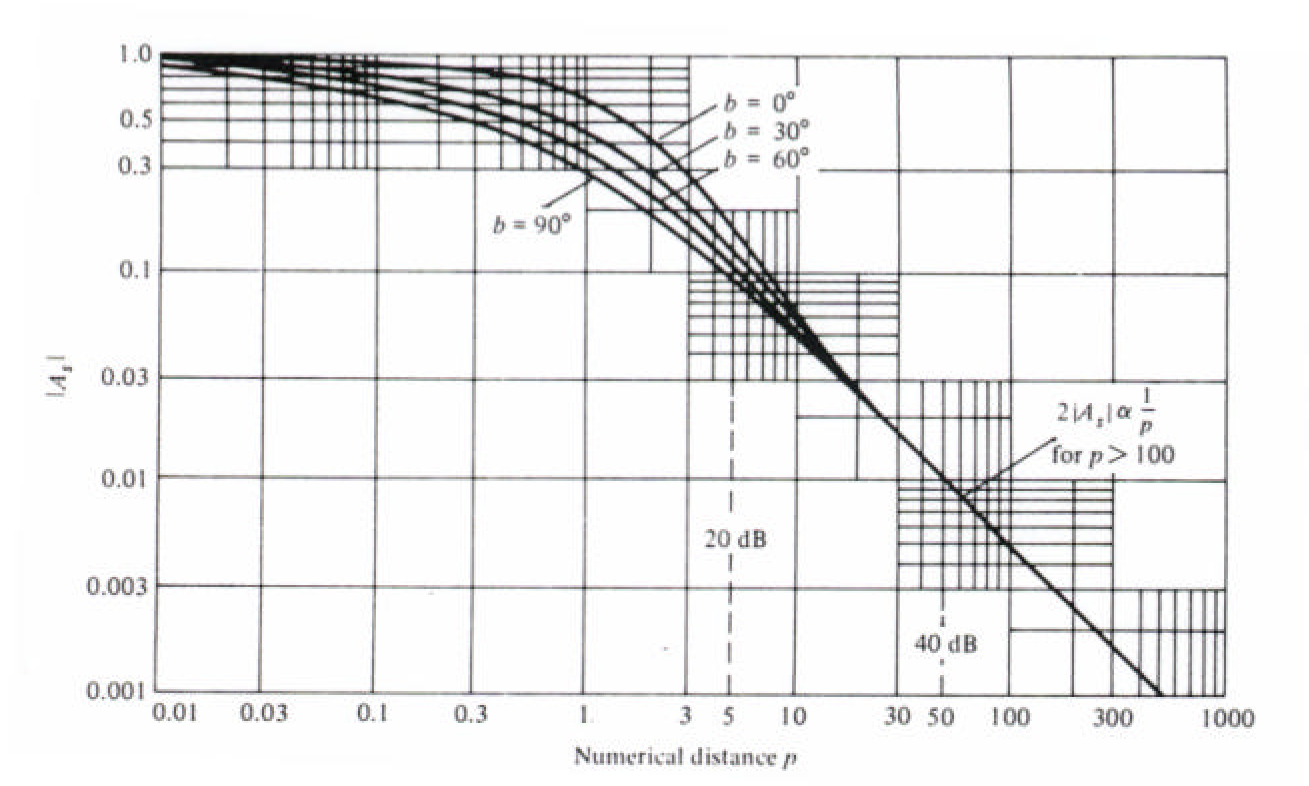


Fig. 6.35 in R. E. Collin, *Antennas and Radiowave Propagation*, McGraw-Hill, 1985

Ground Waves (5)

SPHERICAL EARTH ($e_r = 15$ and $s = 10^{-2}$ S/m)

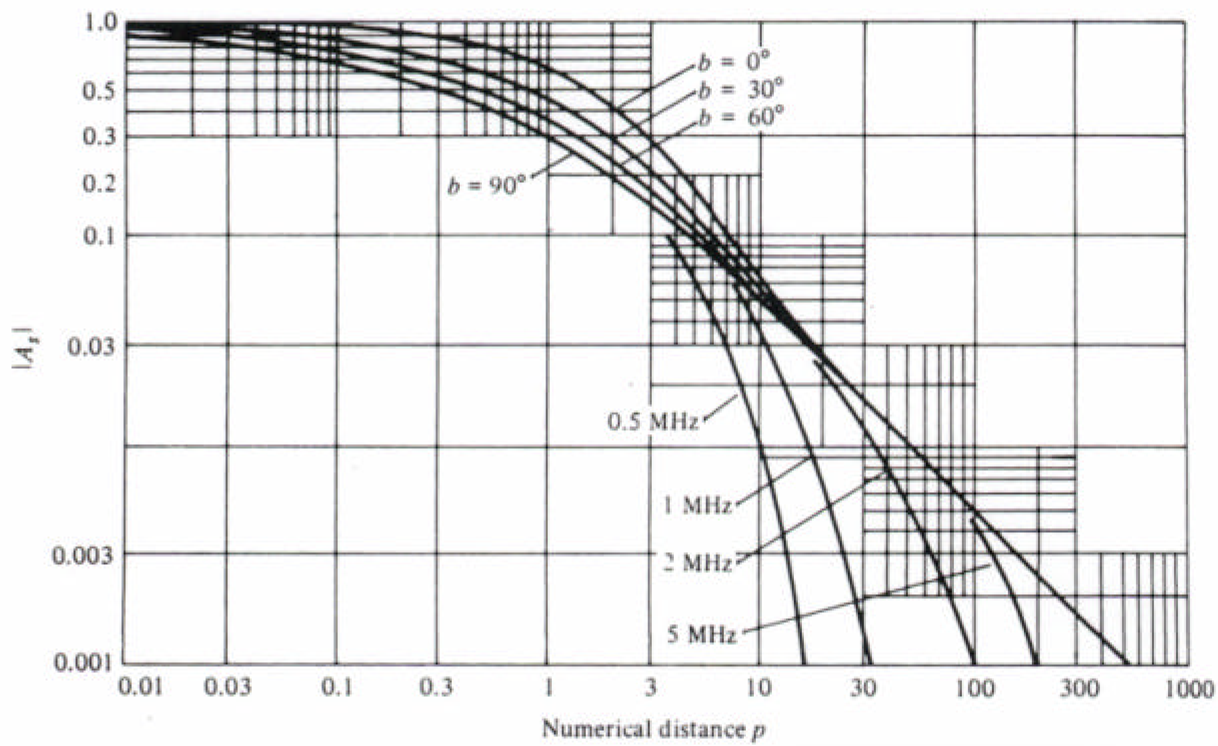
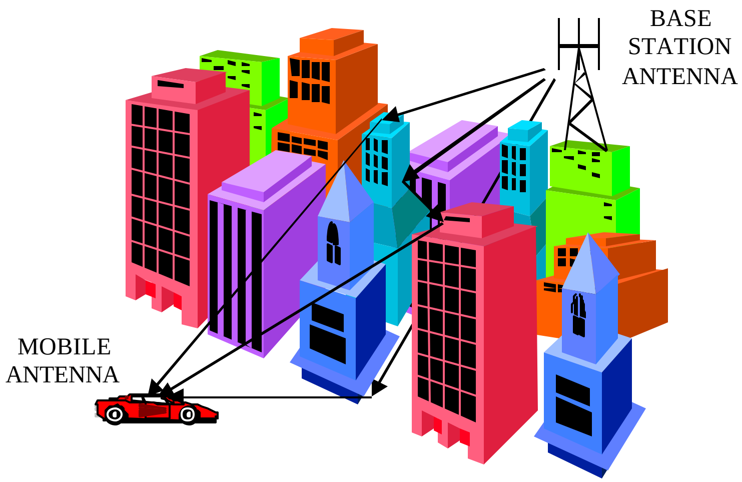


Fig. 6.36 in R. E. Collin, *Antennas and Radiowave Propagation*, McGraw-Hill, 1985

Urban Propagation (1)

Urban propagation is a unique and relatively new area of study. It is important in the design of cellular and mobile communication systems. A complete theoretical treatment of propagation in an urban environment is practically intractable. Many combinations of propagation mechanisms are possible, each with different paths. The details of the environment change from city to city and from block to block within a city. Statistical models are very effective in predicting propagation in this situation.

In an urban or suburban environment there is rarely a direct path between the transmitting and receiving antennas. However there usually are multiple reflection and diffraction paths between a transmitter and receiver.



- Reflections from objects close to the mobile antenna will cause multiple signals to add and cancel as the mobile unit moves. Almost complete cancellation can occur resulting in "deep fades." These small-scale (on the order of tens of wavelengths) variations in the signal are predicted by Rayleigh statistics.

Urban Propagation (2)

- On a larger scale (hundreds to thousands of wavelengths) the signal behavior, when measured in dB, has been found to be normally distributed (hence referred to a lognormal distribution). The genesis of the lognormal variation is the multiplicative nature of shadowing and diffraction of signals along rooftops and undulating terrain.
- The Hata model is used most often for predicting path loss in various types of urban conditions. It is a set of empirically derived formulas that include correction factors for antenna heights and terrain.

Path loss is the $1/r^2$ spreading loss in signal between two isotropic antennas. From the Friis equation, with $G_t = G_r = 4\pi A_e / \lambda^2 = 1$

$$L_s = \frac{P_r}{P_t} = \frac{(1)(1)\,\lambda^2}{(4\pi r)^2} = \left(\frac{1}{2kr}\right)^2$$

Note that path loss is not a true loss of energy as in the case of attenuation. Path loss as defined here will occur even if the medium between the antennas is lossless. It arises because the transmitted signal propagates as a spherical wave and hence power is flowing in directions other than towards the receiver.

Urban Propagation (3)

Hata model parameters*: $d =$ transmit/receive distance ($1 \le d \le 20$ km)

$f =$ frequency in MHz ($100 \le f \le 1500$ MHz)

$h_b =$ base antenna height ($30 \le h_b \le 200$ m)

$h_m =$ mobile antenna height ($1 \le h_m \le 10$ m)

The median path loss is

$$L_{\text{med}} = 69.55 + 26.16\log(f) - 13.82\log(h_b) + \left[44.9 - 6.55\log(h_b)\right]\log(d) + a(h_m)$$

In a medium city: $a(h_m) = \left[0.7 - 1.1\log(f)\right]h_m + 1.56\log(f) - 0.8$

In a large city: $a(h_m) = \begin{cases} 1.1 - 8.29\log^2(1.54h_m), & f \le 200 \text{ MHz} \\ 4.97 - 3.2\log^2(11.75h_m), & f \ge 400 \text{ MHz} \end{cases}$

Correction factors: $L_{\text{cor}} = \begin{cases} -2\log^2(f/28) - 5.4, & \text{suburban areas} \\ -4.78\log^2(f) + 18.33\log(f) - 40.94, & \text{open areas} \end{cases}$

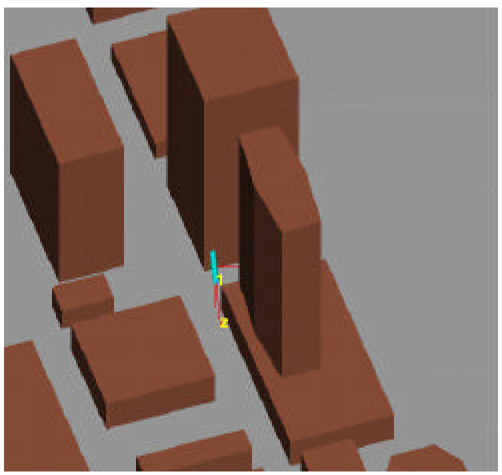
The total path loss is: $L_s = L_{\text{med}} - L_{\text{cor}}$

*Note: Modified formulas have been derived to extend the range of all parameters.

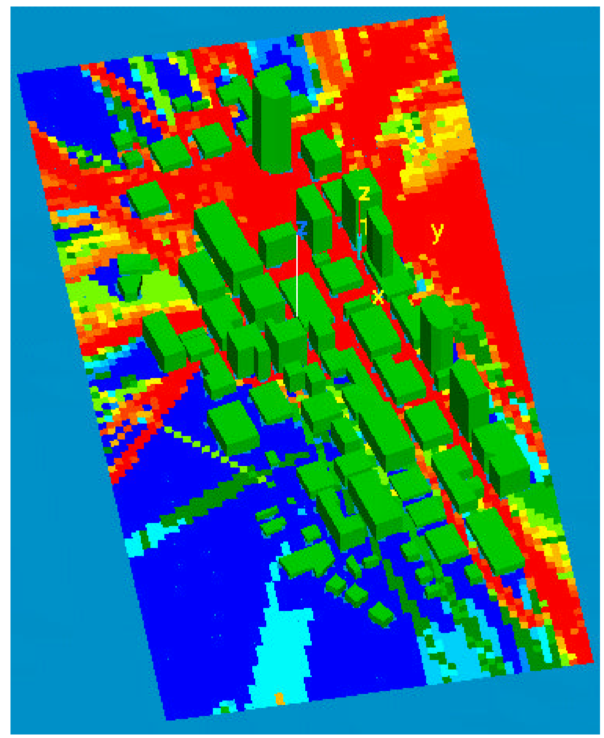
Urban Propagation Simulation

Urban propagation modeling using the wireless toolset *Urbana* (from Demaco/SAIC). Ray tracing (geometrical optics) is used along with the geometrical theory of diffraction (GTD)

Closeup showing antenna placement (below)



Carrier to Interference (C/I) ratio (right)



Measured Data

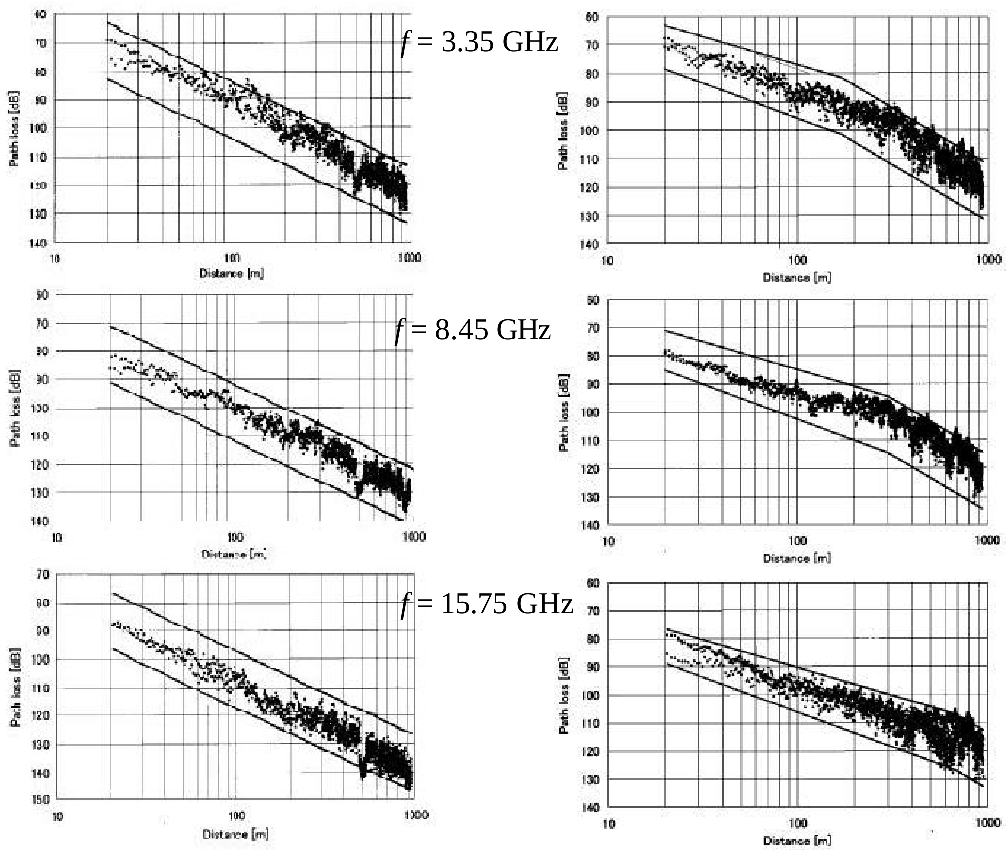
Two different antenna heights

h = 2.7 m h = 1.6 m

f = 3.35 GHz

f = 8.45 GHz

f = 15.75 GHz



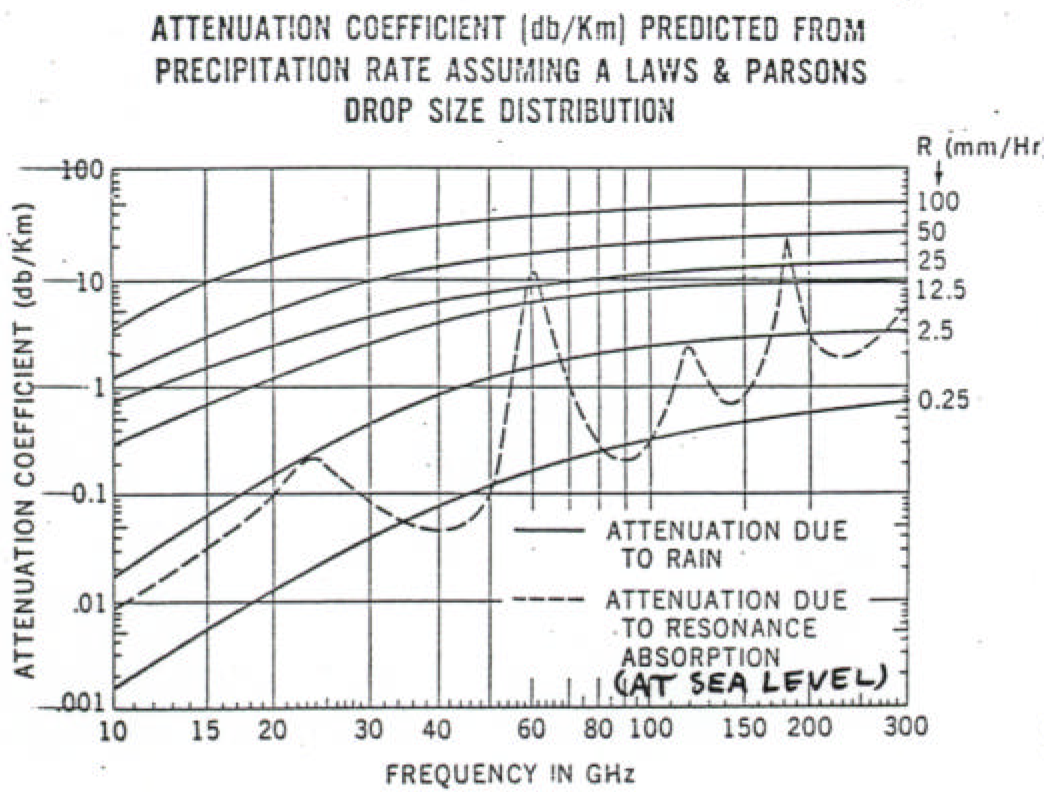
Measured data in an urban environment

Three different frequencies

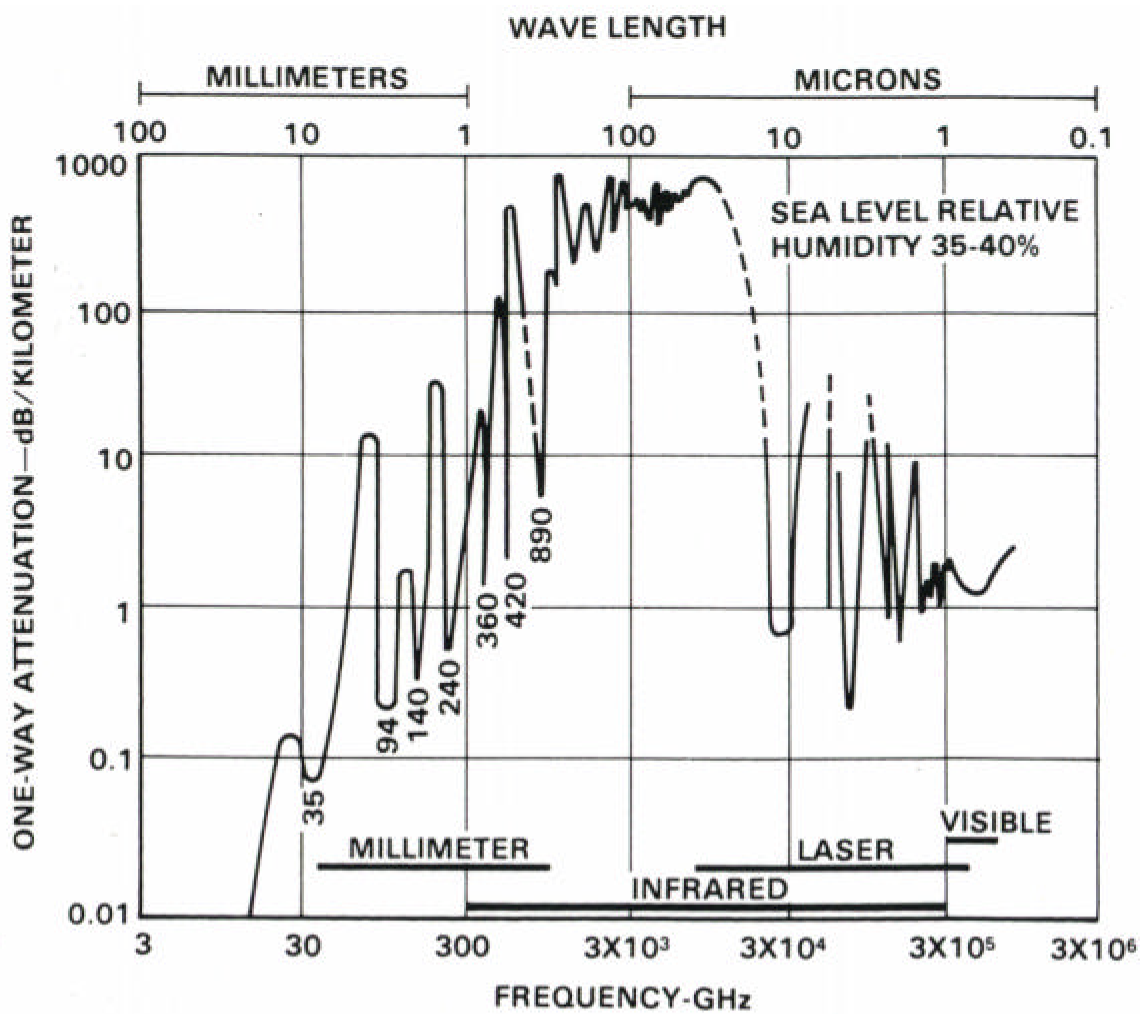
From Masui, "Microwave Path Loss Modeling in Urban LOS Environments," *IEEE Journ. on Selected Areas in Comms*., Vol 20, No. 6, Aug. 2002.

Attenuation Due to Rain and Gases (1)

Sources of signal attenuation in the atmosphere include rain, fog, water vapor and other gases. Most loss is due to absorption of energy by the molecules in the atmosphere. Dust, snow, and rain can also cause a loss in signal by scattering energy out of the beam.



Attenuation Due to Rain and Gases (2)



Attenuation Due to Rain and Gases (3)

There is no complete, comprehensive macroscopic theoretical model to predict loss. A wide range of empirical formulas exist based on measured data. A typical model:

$A = aR^b$, attenuation in dB/km
R is the rain rate in mm/hr

$$a = G_a f_{\mathrm{GHz}}{}^{E_a}$$

$$b = G_b f_{\mathrm{GHz}}{}^{E_b}$$

where the constants are determined from the following table:

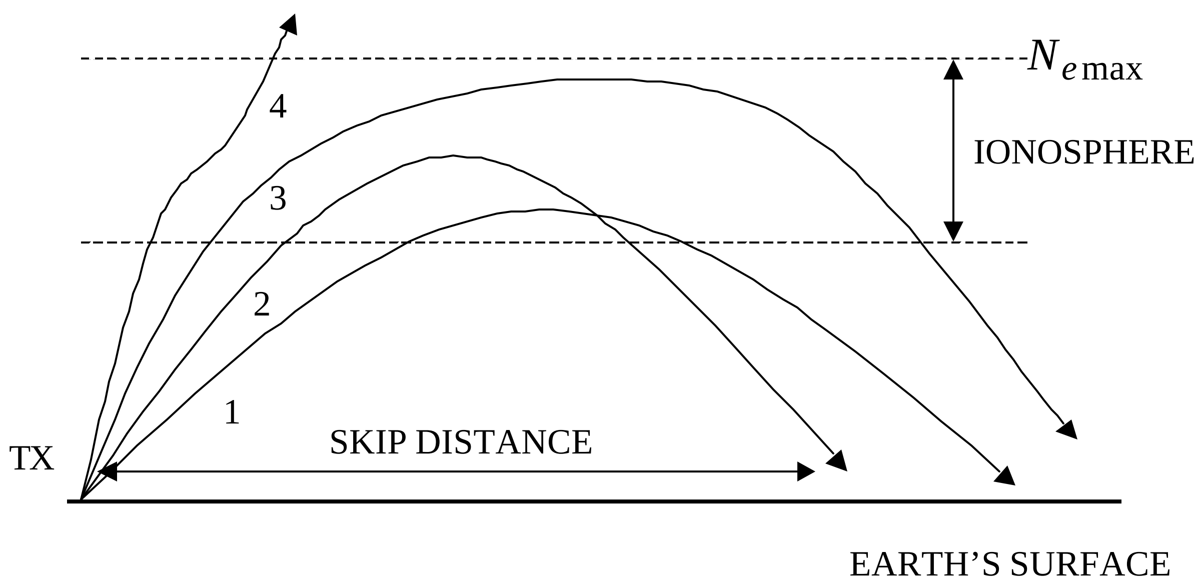
$$\begin{array}{llll}
G_a & = 6.39\times10^{-5} & E_a = 2.03 & f_{\mathrm{GHz}} < 2.9 \\
 & = 4.21\times10^{-5} & \quad\;\, = 2.42 & 2.9 \le f_{\mathrm{GHz}} < 54 \\
 & = 4.09\times10^{-2} & \quad\;\, = 0.699 & 54 \le f_{\mathrm{GHz}} < 180
\end{array}$$

$$\begin{array}{llll}
G_b & = 0.851 & E_b = 0.158 & f_{\mathrm{GHz}} < 8.5 \\
 & = 1.41 & \quad\;\, = -0.0779 & 8.5 \le f_{\mathrm{GHz}} < 25 \\
 & = 2.63 & \quad\;\, = -0.272 & 25 \le f_{\mathrm{GHz}} < 164
\end{array}$$

Ionospheric Radiowave Propagation (1)

The ionosphere refers to the upper regions of the atmosphere (90 to 1000 km). This region is highly ionized, that is, it has a high density of free electrons (negative charges) and positively charged ions. The charges have several important effects on EM propagation:

1. Variations in the electron density (N_e) cause waves to bend back towards Earth, but only if specific frequency and angle criteria are satisfied. Some examples are shown below. Multiple skips are common thereby making global communication possible.



Ionospheric Radiowave Propagation (2)

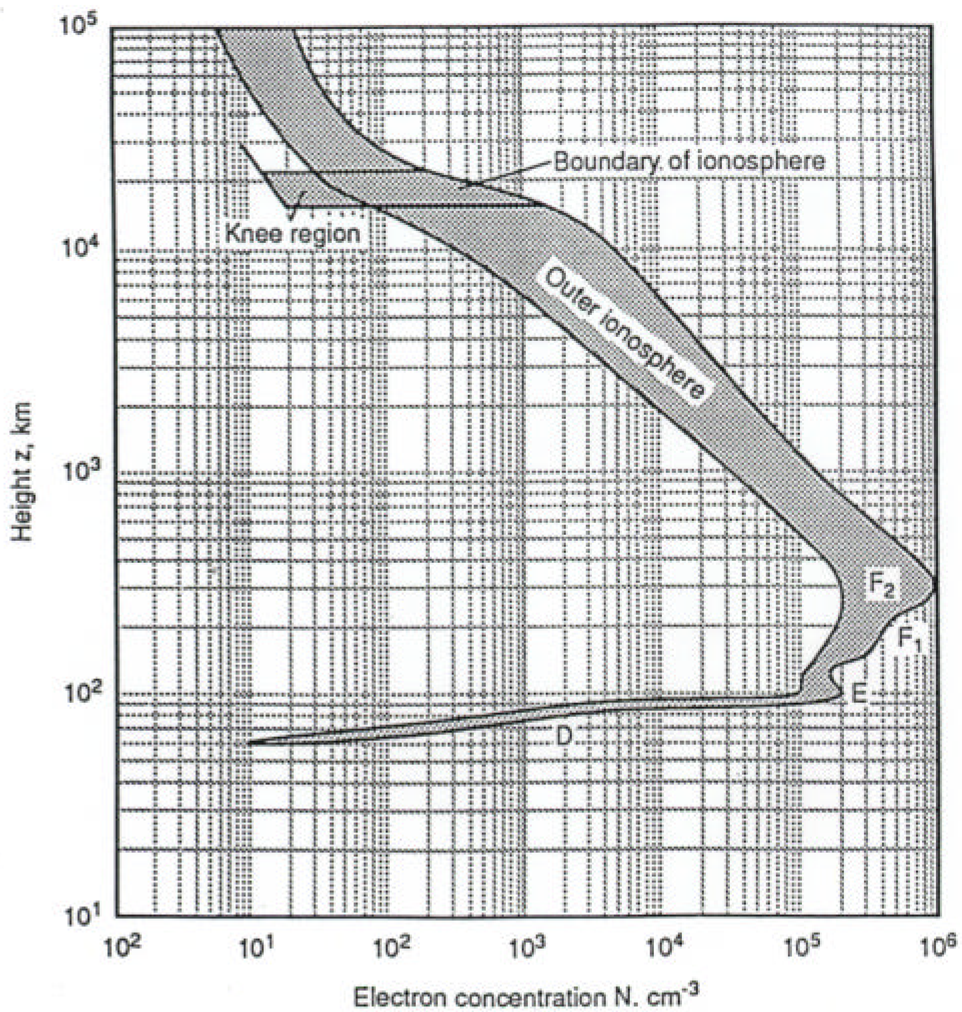
2. The Earth's magnetic field causes the ionosphere to behave like an anisotropic medium. Wave propagation is characterized by two polarizations ("ordinary" and "extra-ordinary" waves). The propagation constants of the two waves are different. An arbitrarily polarized wave can be decomposed into these two polarizations upon entering the ionosphere and recombined on exiting. The recombined wave polarization will be different that the incident wave polarization. This effect is called Faraday rotation.

The electron density distribution has the general characteristics shown on the next page. The detailed features vary with

- location on Earth,
- time of day,
- time of year, and
- sunspot activity.

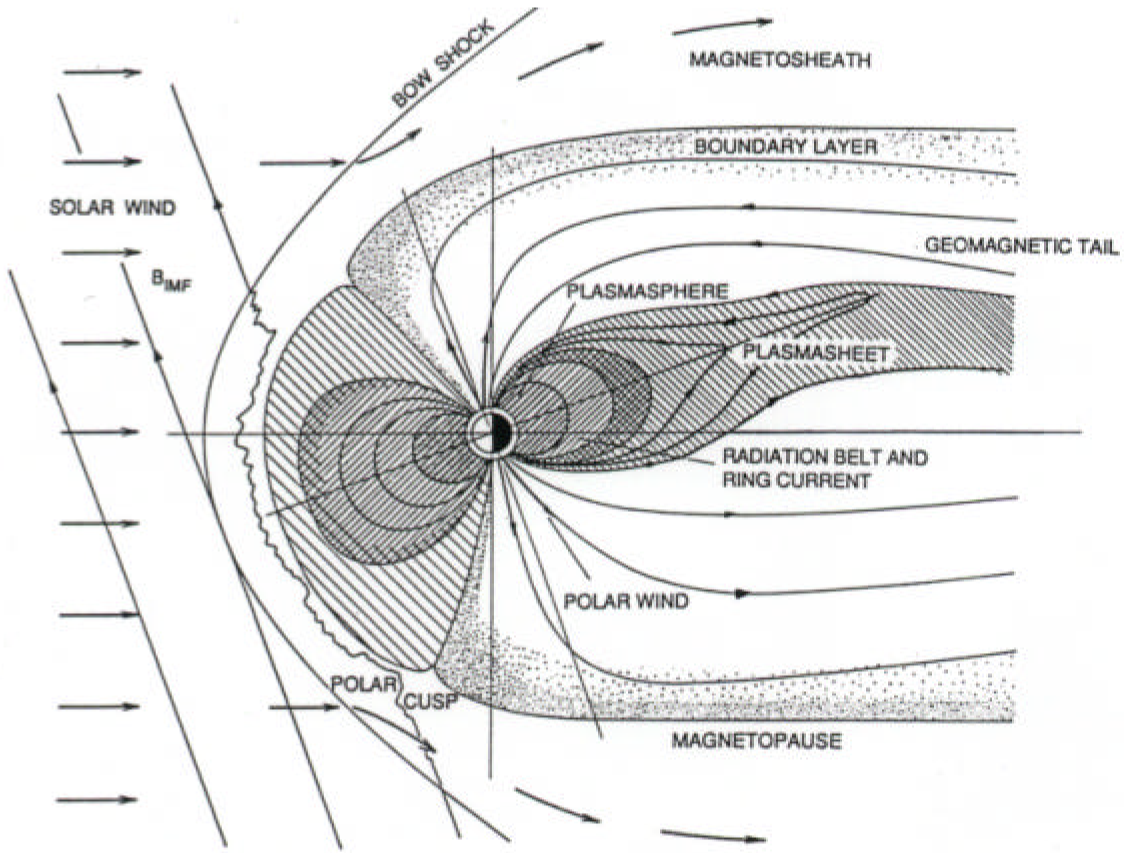
The regions around peaks in the density are referred to as layers. The F layer often splits into the F_1 and F_2 layers.

Electron Density of the Ionosphere



(Note unit is per cubic centimeter)

The Earth's Magnetosphere



Ionospheric Radiowave Propagation (3)

Relative dielectric constant of an ionized gas (assume electrons only):

$$e_r = 1 - \frac{w_p^2}{w(w - jn)}$$

where: n = collision frequency (collisions per second)

$w_p = \sqrt{\frac{N_e e^2}{m e_o}}$, plasma frequency (radians per second)

N_e = electron density ($/\,\mathrm{m}^3$)

$e = 1.59 \times 10^{-19}$ C, electron charge

$m = 9.0 \times 10^{-31}$ kg, electron mass

For the special case of no collions, $n = 0$ and the corresponding propagation constant is

$$k_c = w\sqrt{m_o e_r e_o} = k_o \sqrt{1 - \frac{w_p^2}{w^2}}$$

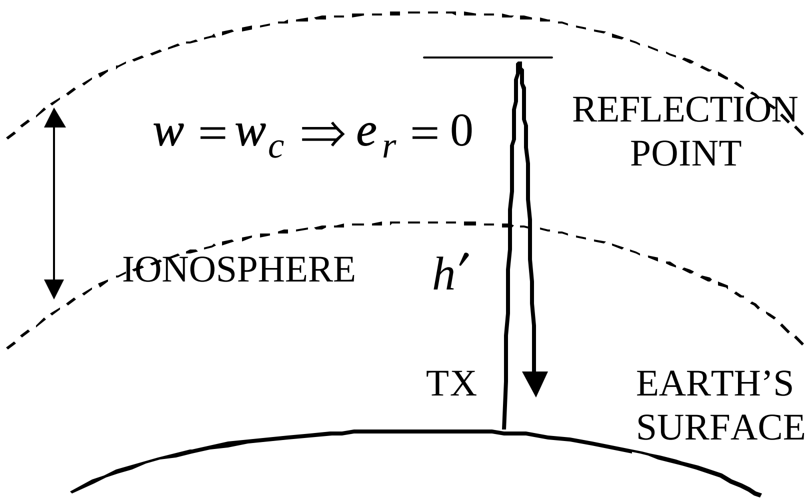
where $k_o = w\sqrt{m_o e_o}$.

Ionospheric Radiowave Propagation (4)

Consider three cases:

1. $w > w_p$: k_c is real and $e^{-jk_c z} = e^{-j|k_c|z}$ is a propagating wave
2. $w < w_p$: k_c is imaginary and $e^{-jk_c z} = e^{-|k_c|z}$ is an evanescent wave
3. $w = w_p$: $k_c = 0$ and this value of w is called the <u>critical frequency</u>, w_c

At the critical frequency the wave is reflected. Note that w_c depends on altitude because the electron density is a function of altitude. For electrons, the highest frequency at which a reflection occurs is



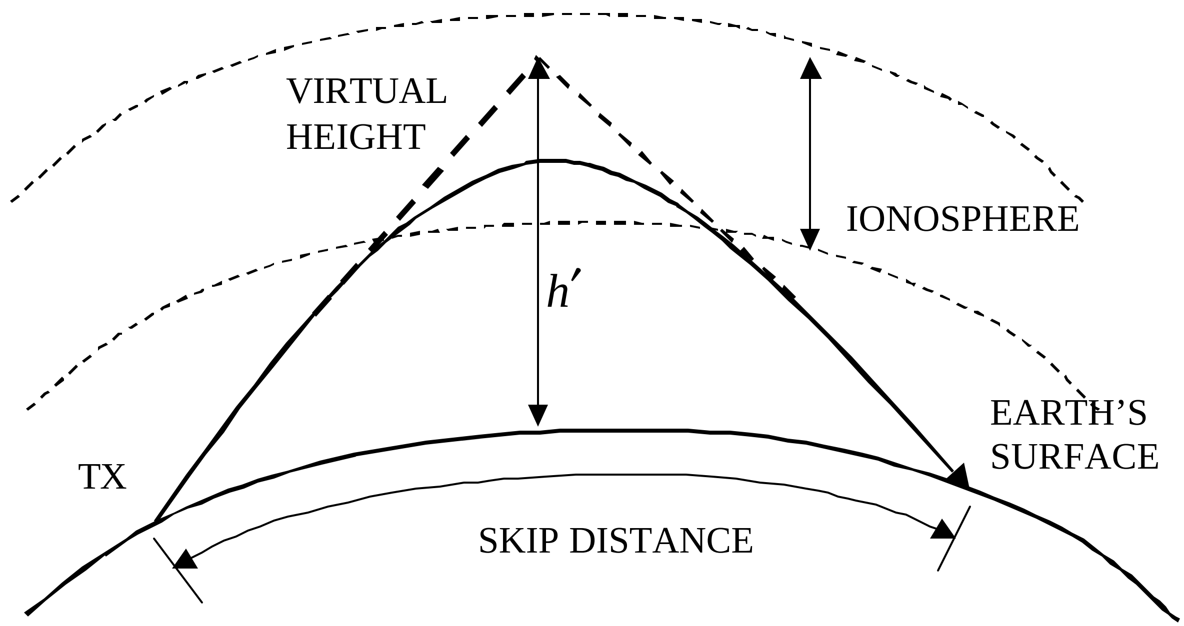
$$f_c = \frac{w_c}{2p} \approx 9\sqrt{N_{e\max}}$$

Reflection at normal incidence requires the greatest N_e.

[1] The critical frequency is where the propagation constant is zero. Neglecting the Earth's magnetic field, this occurs at the plasma frequency, and hence the two terms are often used interchangeably.

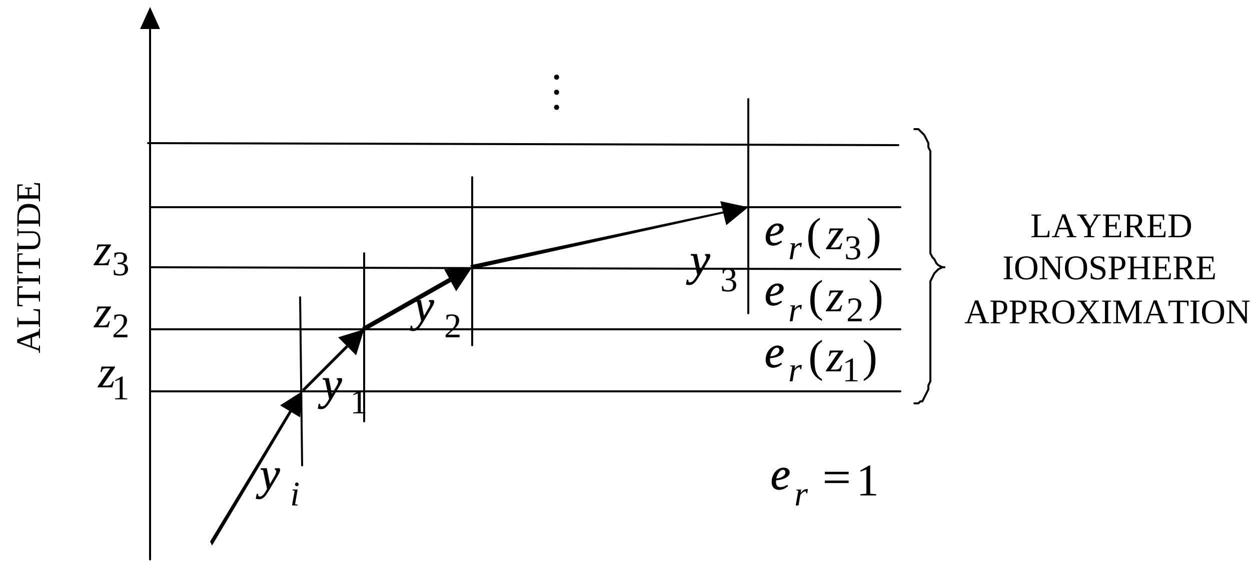
Ionospheric Radiowave Propagation (5)

At oblique incidence, at a point of the ionosphere where the critical frequency is f_c, the ionosphere can reflect waves of higher frequencies than the critical one. When the wave is incident from a non-normal direction, the reflection appears to occur at a virtual reflection point, h', that depends on the frequency and angle of incidence.



Ionospheric Radiowave Propagation (6)

To predict the bending of the ray we use a layered approximation to the ionosphere just as we did for the troposphere.



Snell's law applies at each layer boundary

$$\sin y_i = \sin(y_1)\sqrt{e_r(z_1)} = \cdots$$

The ray is turned back when $y(z) = p/2$, or $\sin y_i = \sqrt{e_r(z)}$

Ionospheric Radiowave Propagation (7)

Note that:

1. For constant y_i, N_e must increase with frequency if the ray is to return to Earth (because e_r decreases with w).
2. Similarly, for a given maximum N_e ($N_{e\,\max}$), the maximum value of y_i that results in the ray returning to Earth increases with increasing w.

<u>There is an upper limit on frequency that will result in the wave being returned back to Earth</u>. Given $N_{e\,\max}$ the required relationship between y_i and f can be obtained

$$\sin y_i = \sqrt{e_r(z)}$$

$$\sin^2 y_i = 1 - \frac{w_p^2}{w^2}$$

$$1 - \cos^2 y_i = 1 - \frac{81 N_{e\,\max}}{f^2}$$

$$N_{e\,\max} = \frac{f^2 \cos^2 y_i}{81} \quad \Rightarrow \quad f_{\max} = \sqrt{\frac{81 N_{e\,\max}}{\cos^2 y_i}}$$

Ionospheric Radiowave Propagation (8)

Examples:

1. $y_i = 45^\circ,\ N_{e\max} = 2\times 10^{10}\ /\,\mathrm{m}^3$: $\ f_{\max} = \sqrt{(81)(2\times 10^{10})/(0.707)^2} = 1.8\ \text{MHz}$
2. $y_i = 60^\circ,\ N_{e\max} = 2\times 10^{10}\ /\,\mathrm{m}^3$: $\ f_{\max} = \sqrt{(81)(2\times 10^{10})/(0.5)^2} = 2.5\ \text{MHz}$

The value of f that makes $e_r = 0$ for a given value of $N_{e\max}$ is the critical frequency defined earlier:

$$f_c = 9\sqrt{N_{e\max}}$$

Use the $N_{e\max}$ expression from previous page and solve for f

$$f = 9\sqrt{N_{e\max}}\sec y_i = f_c \sec y_i$$

This is called the <u>secant law</u> or <u>Martyn's law</u>. When $\sec y_i$ has its maximum value, the frequency is called the <u>maximum usable frequency</u> (MUF). A typical value is less than 40 MHz. It can drop as low as 25 MHz during periods of low solar activity. The <u>optimum usable frequency</u> (OUF) is 50% to 80% of the MUF.

Maximum Usable Frequency

The maximum usable frequency (MUF) in wintertime for different skip distances. The MUF is lower in the summertime.

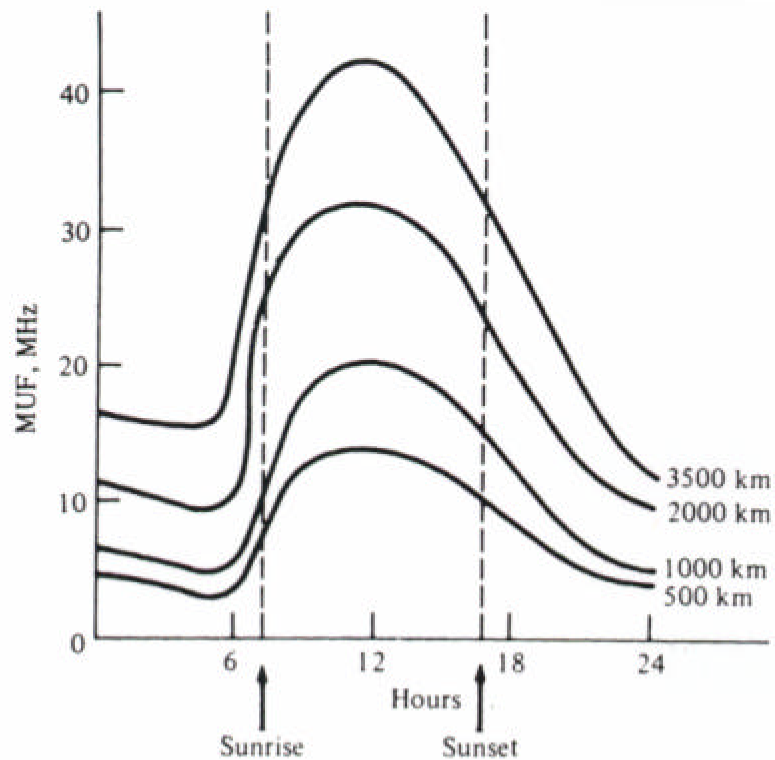
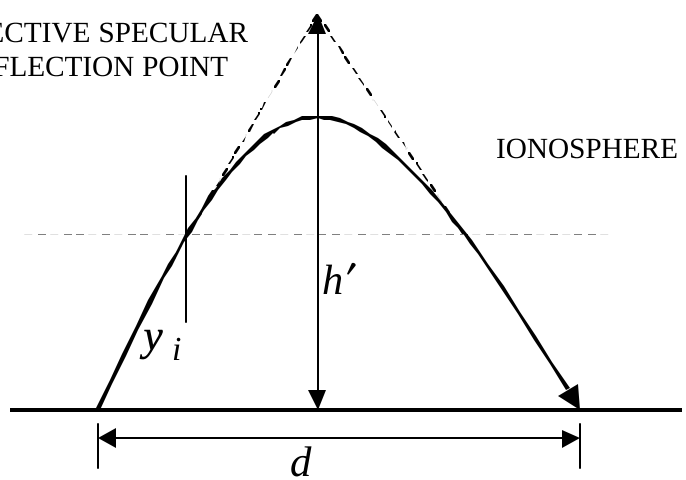


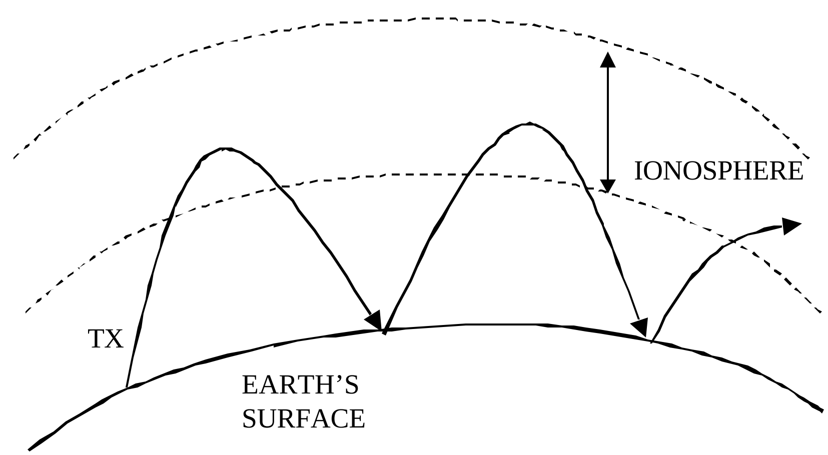
Fig. 6.43 in R. E. Collin, *Antennas and Radiowave Propagation*, McGraw-Hill, 1985

Ionospheric Radiowave Propagation (9)

Multiple hops allow for very long range communication links (transcontinental). Using a simple flat Earth model, the virtual height (h'), incidence angle (ψ_i), and skip distance (d) are related by $\tan\psi_i = \frac{d}{2h'}$. This implies that the wave is launched well above the horizon. However, if a spherical Earth model is used and the wave is launched on the horizon then $d = 2\sqrt{2R_e'h'}$.



Single ionospheric hop
(flat Earth)



Multiple ionospheric hops
(curved Earth)

Ionospheric Radiowave Propagation (10)

Approximate virtual heights for layers of the ionosphere

Layer	Range for h' (km)
F_2	250 to 400 (day)
F_1	200 to 250 (day)
F	300 (night)
E	110

Example: Based on geometry, a rule of thumb for the maximum incidence angle on the ionosphere is about 74°. The MUF is

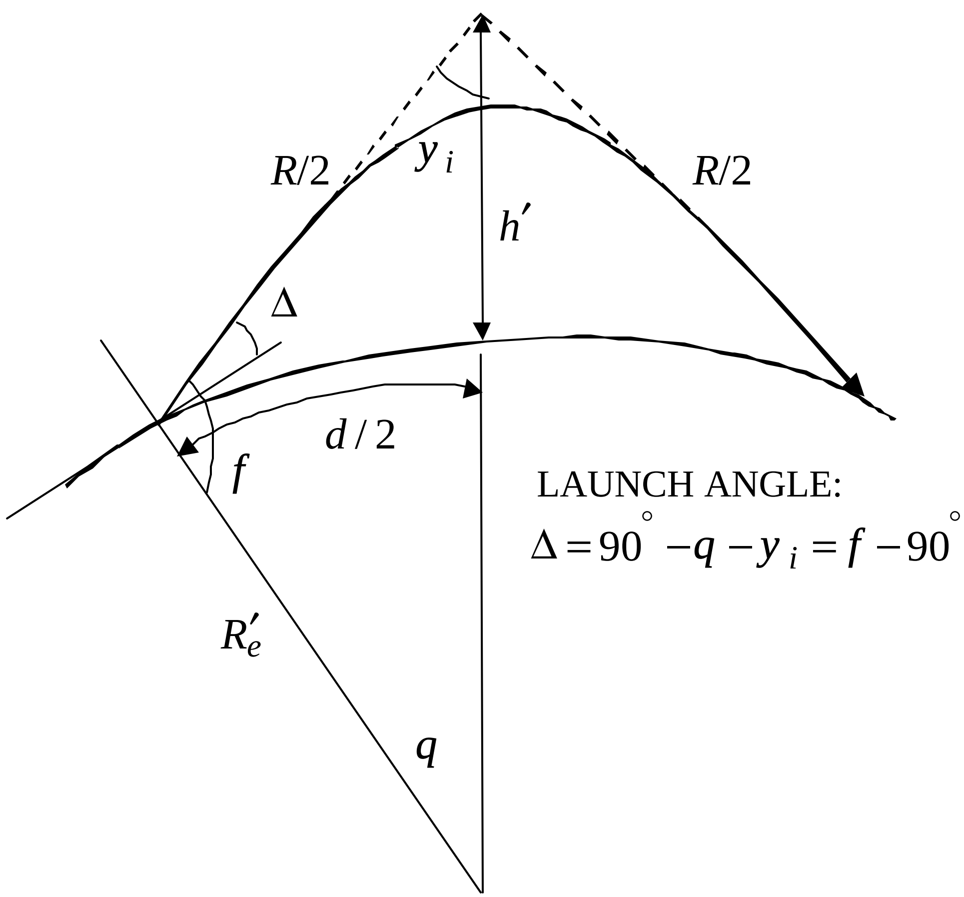
$$\text{MUF} = f_c \sec(74^\circ) = 3.6 f_c$$

For $N_{e\max} = 10^{12}\ /\text{m}^3$, $f_c \approx 9$ MHz and the $\text{MUF} = 32.4$ MHz. For reflection from the F_2 layer, $h' \approx 300$ km. The maximum skip distance will be about

$$d_{\max} \approx 2\sqrt{2R_e' h'} = 2\sqrt{2(8500\times 10^3)(300\times 10^3)} = 4516 \text{ km}$$

Ionospheric Radiowave Propagation (7)

For a curved Earth, using the law of sines for a triangle $\dfrac{1+h'/R_e' - \cos q}{\sin q} = \dfrac{1}{\tan y_i}$



where

$$q = \frac{d}{2R_e'}$$

and the launch angle (antenna pointing angle above the horizon) is

$$\Delta = f - 90^\circ = 90^\circ - q - y_i$$

The great circle path via the reflection point is R, which can be obtained from

$$R = \frac{2R_e' \sin q}{\sin y_i}$$

Ionospheric Radiowave Propagation (8)

Example: Ohio to Europe skip (4200 miles = 6760 km). Can it be done in one hop?

To estimate the hop, assume that the antenna is pointed on the horizon. The virtual height required for the total distance is

$$d/2 = R_e' q \quad \rightarrow \quad q = d/\left(2R_e'\right) = 0.3976 \text{ rad} = 22.8 \text{ degrees}$$
$$(R_e' + h')\cos q = R_e' \quad \rightarrow \quad h' = R_e'/\cos q - R_e' = 720 \text{ km}$$

This is above the F layer and therefore two skips must be used. Each skip will be half of the total distance:. Repeating the calculation for $d/2 = 1690$ km gives

$$q = d/\left(2R_e'\right) = 0.1988 \text{ rad} = 11.39 \text{ degrees}$$
$$h' = R_e'/\cos q - R_e' = 171 \text{ km}$$

This value lies somewhere in the F layer. We will use 300 km (a more typical value) in computing the launch angle. That is, still keep $d/2 = 1690$ km and $q = 11.39$ degrees, but point the antenna above the horizon to the virtual reflection point at 300 km

$$\tan y_i = \sin(11.39^\circ)\left[1 + \frac{300}{8500} - \cos(11.39^\circ)\right]^{-1} \quad \rightarrow \quad y_i = 74.4^\circ$$

Ionospheric Radiowave Propagation (9)

The actual launch angle required (the angle that the antenna beam should be pointed above the horizon) is

$$\text{launch angle, } \Delta = 90^\circ - q - y_i = 90^\circ - 11.39^\circ - 74.4^\circ = 4.21^\circ$$

The electron density at this height (see chart, p.3) is $N_{e\max} \approx 5\times10^{11}\ /\mathrm{m}^3$ which corresponds to the critical frequency

$$f_c \approx 9\sqrt{N_{e\max}} = 6.36 \text{ MHz}$$

and a MUF of

$$\text{MUF} \approx 6.36 \sec 74.4^\circ = 23.7 \text{ MHz}$$

Operation in the international short wave 16-m band would work. This example is oversimplified in that more detailed knowledge of the state of the ionosphere would be necessary: time of day, time of year, time within the solar cycle, etc. These data are available from published charts.

Ionospheric Radiowave Propagation (10)

Generally, to predict the received signal a modified Friis equation is used:

$$P_r = \frac{P_t G_t G_r}{\left(4pR / l\right)^2} L_x L_a$$

where the losses, in dB, are negative:

$L_x = L_{\mathrm{pol}} + L_{\mathrm{refl}} - G_{\mathrm{iono}}$

L_{refl} = reflection loss if there are multiple hops

L_{pol} = polarization loss due to Faraday rotation and earth reflections

G_{iono} = gain due to focussing by the curvature of the ionosphere

L_a = absorption loss

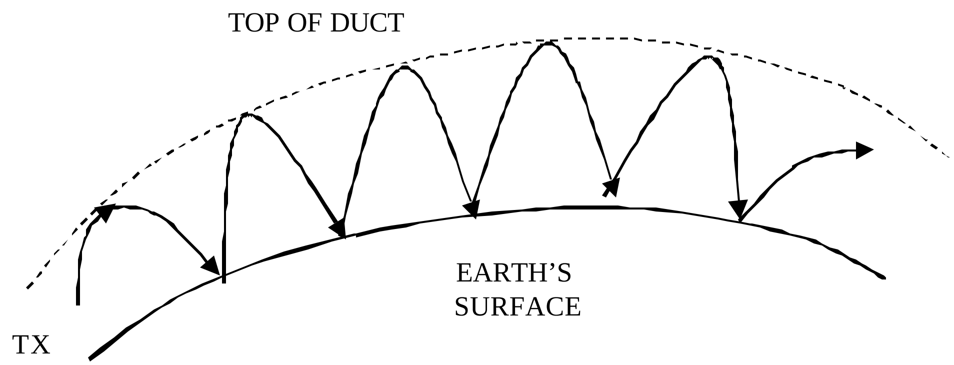
R = great circle path via the virtual reflection point

Example: For $P_t = 30$ dBW, f = 10 MHz, $G_t = G_r = 10$ dB, d = 2000 km, $h' = 300$ km, $L_x = 9.5$ dB and $L_a = 30$ dB (data obtained from charts).

From geometry compute: $y_i = 70.3^\circ$, R = 2117.8 km, and thus $P_r = -108.5$ dBw

Ducts and Nonstandard Refraction (1)

Ducts in the atmosphere are caused by index of refraction rates of decrease with height over short distances that cause rays to bend back towards the surface.



- The formation of ducts is due primarily to water vapor, and therefore they tend to occur over bodies of water (but not land-locked bodies of water)
- They can occur at the surface or up to 5000 ft (elevated ducts)
- Thickness ranges from a meter to several hundred meters
- The trade wind belts have a more or less permanent duct of about 1 to 5 m thickness
- Efficient propagation occurs for UHF frequencies and above if both the transmitter and receiver are located in the duct
- If the transmitter and receiver are not in the duct, significant loss can occur before coupling into the duct

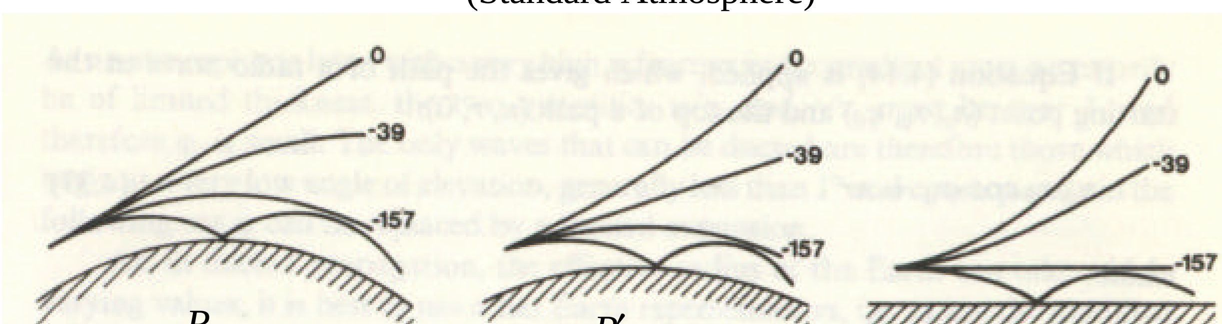
Ducts and Nonstandard Refraction (2)

Because variations in the index of refraction are so small, a quantity called the refractivity is used

$$N(h) = [n(h) - 1]10^6 \qquad n(h) = \sqrt{e_r(h)}$$

In the normal (standard) atmosphere the gradient of the vertical refractive index is linear with height, $dN/dh \approx -39$ N units/km. If $dN/dh < -157$ then rays will return to the surface. Rays in the three Earth models are shown below.

True Earth | Equivalent Earth (Standard Atmosphere) | Flat Earth



R_e | R_e' | ∞

From *Radiowave Propagation*, Lucien Boithias, McGraw-Hill

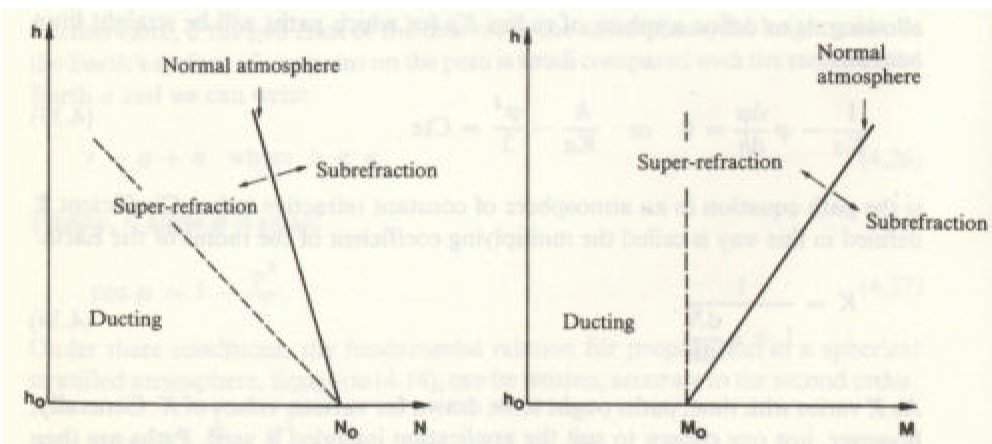
Ducts and Nonstandard Refraction (3)

Another quantity used to solve ducting problems is the modified refractivity

$$M(h) = N(h) + 10^6 (h / R'_e)$$

In terms of *M*, the condition for ducting is $dM/dh = dN/dh + 157$. Other values of dN/dh (or dM/dh) lead to several types of refraction as summarized in the following figure and table. They are:

1. Super refraction: The index of refraction decrease is more rapid than normal and the ray curves downward at a greater rate
2. Substandard refraction (subrefraction): The index of refraction decreases less rapidly than normal and there is less downward curvature than normal



From *Radiowave Propagation*, Lucien Boithias, McGraw-Hill

Ducts and Nonstandard Refraction (4)

Summary of refractivity and ducting conditions

<table>
<tr><th>dN / dh</th><th>Ray Curvature</th><th>k</th><th>Atmospheric Refraction</th><th>Virtual Earth</th><th>Horizontally Launched Ray</th></tr>
<tr><td>> 0</td><td>up</td><td>< 1</td><td rowspan="3">below normal</td><td>more convex</td><td rowspan="5">moves away from Earth</td></tr>
<tr><td>0</td><td>none</td><td>1</td><td>actual</td></tr>
<tr><td>$0 > \frac{dN}{dh} > -39$</td><td rowspan="5">down</td><td>> 1</td><td rowspan="3">less convex</td></tr>
<tr><td>-39</td><td>4/3</td><td>normal</td></tr>
<tr><td>$-39 > \frac{dN}{dh} > -157$</td><td>$> 4/3$</td><td rowspan="2">above normal</td></tr>
<tr><td>-157</td><td></td><td>plane</td><td>parallel to Earth</td></tr>
<tr><td>< -157</td><td></td><td>super-refraction</td><td>concave</td><td>draws closer to Earth</td></tr>
</table>